A semi-analytical model for near source ground dispersion

16 GMU Conference

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Goals

To construct a robust model for dispersion from continuous near ground sources for distances up to a few hundreds meters:

- Properties
  - takes into account the variation of the average wind velocity and the turbulence with height
  - computationally efficient - suitable for repeated computations
- Applications
  - short range concentrations
  - source determination - inverse problem
  - near source concentration in Eulerian dispersion models

$$\text{The choice} \Longrightarrow \text{A semi analytical model}$$

existing analytical models

Most common existing analytical models are Gaussian models:

$$C(x,y,z) = q \cdot C_y(x,y) \cdot C_z(x,z)$$

$$C_y(x,y) = \frac{1}{\sqrt{2\pi}\sigma_y(x)} \exp\left[-\left(y/2\sigma_y\right)^2 \right]$$

$$C_z(x,z) = \sqrt{\frac{2}{\pi}} \frac{1}{U\sigma_z(x)} \exp\left[-\left(z/2\sigma_z\right)^2 \right]$$

Properties:

- U is constant
- $\sigma_y(x)$ & $\sigma_z(x)$ are independent of Z

Unrealistic limitations for short range ground dispersion

skip - vertical

proposed model assumptions:

- vertical dispersion may be described as a diffusion process - K theory
- lateral dispersion is Gaussian
- for given x, σ_y is uniform in z
- σ_y depends on the height of the cloud center of mass $Z_{com}(x)$

vertical time scales

According to (Hunt & Weber 1979) the time t that takes a cloud to reach z

$$t \approx 10\,T_L(z) \quad \Longrightarrow \quad \text{diffusive limit}$$

$$\Longrightarrow \quad \text{vertical solution based on K theory}$$

TL

solution structure

$$C(x,y,z) = q \cdot C_y(x, Z_{com}(x), y) \cdot C_z(x,z)$$

$$C_y(x, Z_{com}(x), y) = \frac{1}{\sqrt{2\pi}\sigma_y} \exp\left[-\left(y/2\sigma_y\right)^2 \right]$$

$C_z(x,z)$ is a solution of a non-Gaussian diffusion-advection equation

[see for example Sutton(1943), Sutton(1953), Huang(1979), Arya(1999), Brown et al (1993)]

For the lateral structure $C_y(x,y)$, σ_y along the $C.O.M.$ should be determined

To this end:

1. The c.o.m. path $Z_{com}(x)$ should be determined
2. One needs an equation for the change of $\sigma_y(x)$ along $Z_{com}(x)$

vertical equation

$$U(z)\frac{\partial C_z}{\partial x} = \frac{\partial}{\partial z}\left(K(z)\frac{\partial C_z}{\partial z}\right)$$

$$U(z) = az^m$$
$$K(z) = bz^n$$

a & m may be obtained directly from measurements

b & n should be deduced from theory (similarity)

vertical solution - Sutton (1943)

$$C_z(x,z) = A \cdot x^{-\beta} \exp\left[-z^{\alpha}/Bx\right]$$
$$K\frac{\partial C_z}{\partial z}|_{z=0} = \delta(x)$$
$$0 < \alpha < 2; \quad \beta > 0$$

vertical solution - b & n parameters

b & n are detrmined by optimization with respect to MOST

$$n = \begin{cases} 1 + \frac{\alpha_1 Z_c}{2(1+\alpha_1 Z_c)} & Z/L \leq 0 \\ \frac{1}{1+\alpha_2 Z_c} & Z/L \geq 0 \end{cases}$$

$$b = \begin{cases} \kappa u_* \frac{1}{Z_c^{n-1}} \sqrt{1+\alpha_1 Z_c} & Z/L \leq 0 \\ \kappa u_* \frac{1}{Z_c^{n-1}} \frac{1}{1+\alpha_2 Z_c} & Z/L \geq 0 \end{cases}$$

$$\alpha_1 = 16/|L|; \quad \alpha_2 = 5/L$$

Z_c is related to the height H - the optimization interval

$$0.61 \approx e^{-1/2} \leq \frac{Z_c}{H} \leq e^{-2/5} \approx 0.67 \qquad Z/L \leq 0$$

$$\exp\left[-\frac{1+0.61\alpha_2 H}{2+0.61\alpha_2 H}\right] \leq \frac{Z_c}{H} \leq \exp\left[-\frac{1+0.35\alpha_2 H}{2+0.35\alpha_2 H}\right] \qquad Z/L \geq 0$$

$H = 1m$ is optimal for distance $\approx 100m$

Lateral Dispersion

$$C_y(x, Z_{com}, y) = \frac{1}{\sqrt{2\pi}\sigma_y(x)} \exp\left[-\left(y/2\sigma_y(x)\right)^2 \right]$$

lateral dispersion - σ_y: assumptions

- The longitudinal velocity along the path $Z_{com}(x)$ is given by the power law $u = a \cdot Z_{com}^m$
- At distance x from the source σ_y depends on the turbulence at the height of the c.o.m. path $z = Z_{com}(x)$.
- σ_y may be determined from a one dimensional Langevin equation for the lateral velocity $\mathbf{V}$

skip - to final

lateral dispersion: cloud width

$$\frac{1}{2}\frac{d}{dt}\sigma_y^2(t) = \int_0^t \overline{v(t)v(s)}ds$$

v is determined from Langevin equation at $z = Z_{com}(x)$:

$$dv = a(z, v, t)dt + b(z, t)d\zeta(t)$$
$$dy = vdt$$

$$< d\zeta(t), d\zeta(s) >= \delta(t - s)$$

$$a(z) = -\frac{1}{2}\frac{b^2}{\sigma_v^2}v = -\frac{1}{T_L(z)}v; \qquad b(z) = \sqrt{C_0\epsilon(z)}$$

final equations for σ_y

$$\frac{d}{dx}\sigma_y^2 = \frac{2\sigma_v^2}{\phi(x)} f; \qquad \sigma_y^2\big|_{x=0} = 0$$

$$\frac{d}{dx} f = \frac{1}{\phi(x)}\Big[1 - \frac{1}{T_L(x)} f\Big]; \quad f\Big|_{x=0} = 0$$

Both $\phi(x)$ and $T_L(x)$ are evaluated at the C.O.M. height.

$$Z_{com}(x) = \frac{\int_0^\infty z C_z(x,z)dz}{\int_0^\infty C_z(x,z)dz} = (Bx)^{1/\alpha}\frac{\Gamma(2/\alpha)}{\Gamma(1/\alpha)}$$

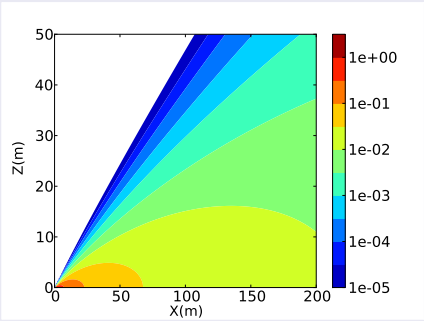
$$\phi(x) \equiv \frac{dX}{dt}\Big|_{z=Z_{com}(x)} = a\big[Z_{com}(x)\big]^m; \quad m < 1$$

$$T_L(x) = \frac{\sigma_v^2}{C_0\epsilon}\Big|_{z=Z_{com}(x)}$$

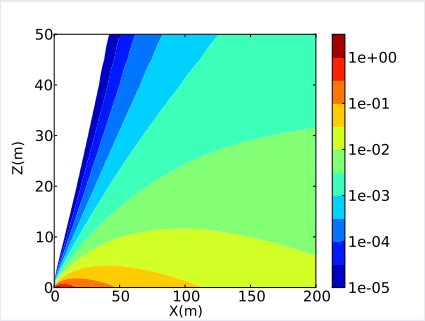
verification & comparison

- verification
  1. vertical solution - calculated c.w.i.c. against similarity solutions obtained from a numerical model
     i. verification of the formulae for the coefficients b & n
     ii. verification of the parameter $H = 1m$
  2. ground concentrations against Flesch et al (2004)
  3. ground concentrations against Prairie Grass experiments
- comparison with a Gaussian model

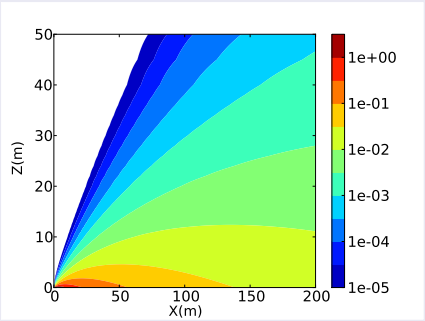
c.w.i.c. comparison; $0 \leq Z \leq 50m$: ▸ skip - to exp



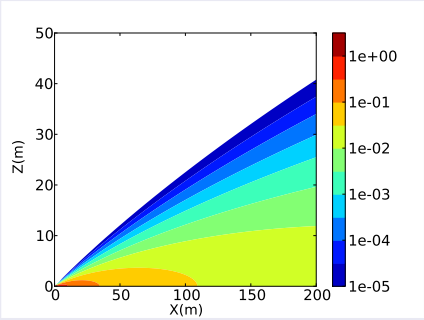
(a) B3 gaussian



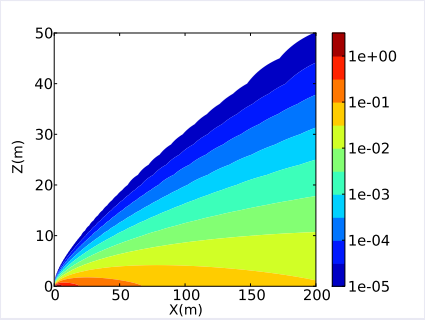
(b) B3 sim



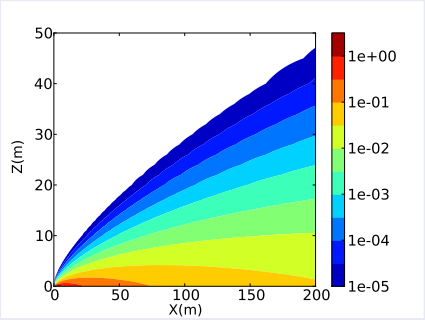
(c) B3 power



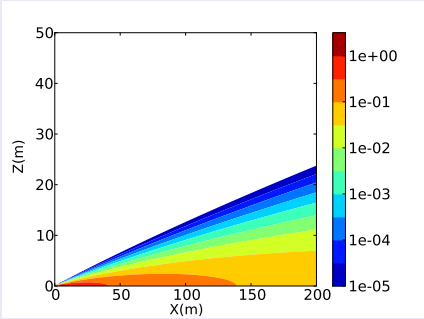
(d) D4 gaussian



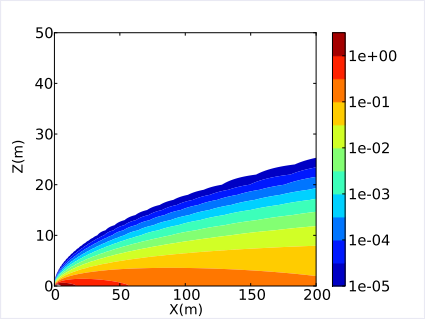
(e) D4 sim



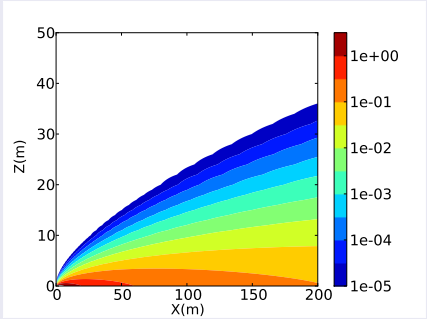
(f) D4 power



(g) E2 gaussian

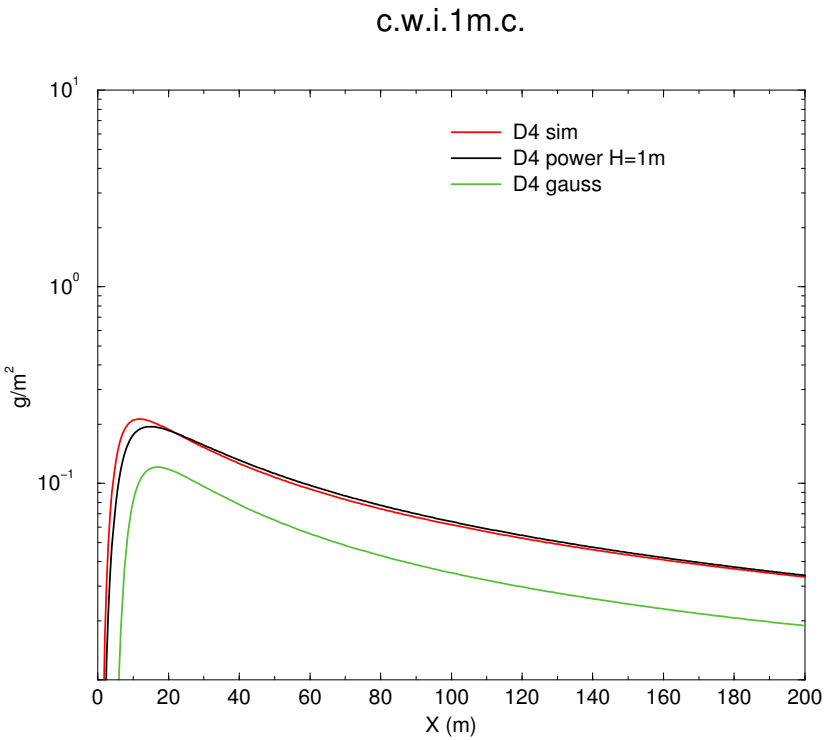


(h) E2 sim

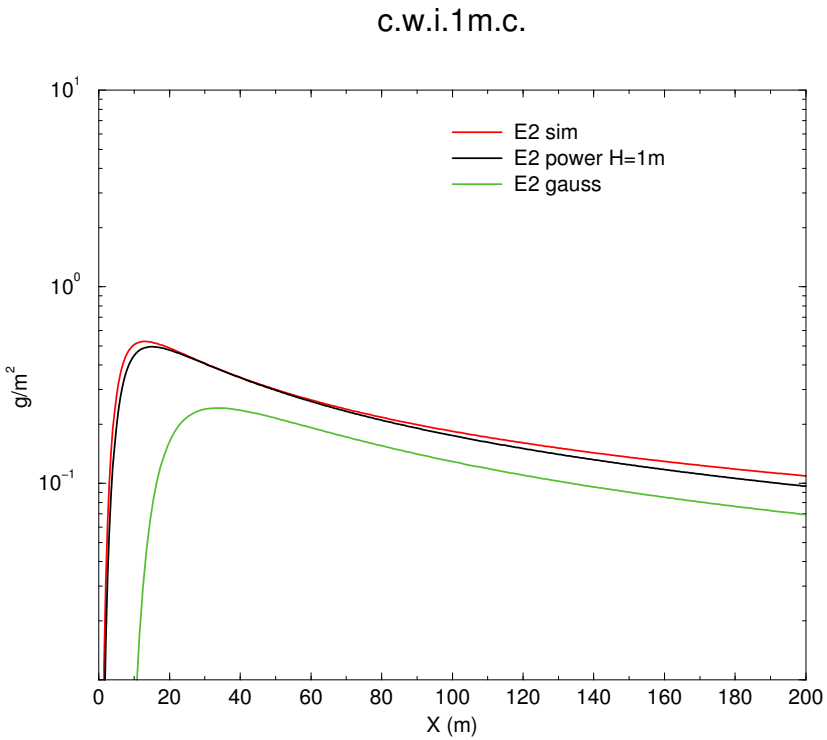


(i) E2 power

c.w.i.c. comparison; Z=1m: power, similarity, Gaussian stable & neutral conditions

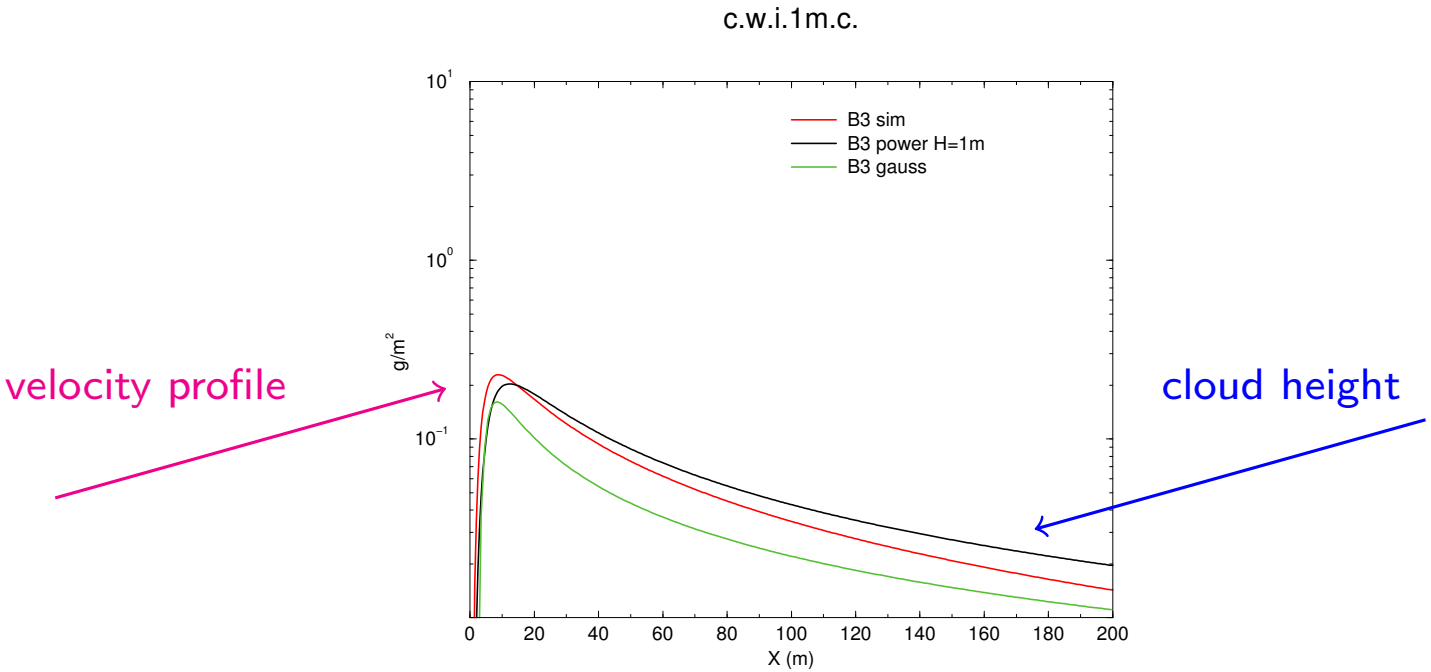


(a) D4 - 1m



(b) E2 - 1m

c.w.i.c. - unstable conditions B3



- power law fails to represent unstable and neutral similarity velocity profiles near the ground ($n + m > 1 \longrightarrow$ violation of constant flux)
- cloud center rises fast - choose $H > 1m$ for $X > 100m$

Flesch et al 2004: Experiment setup

source area 6mX6m; laser path at $\approx h = 1m$; $T = 15min$

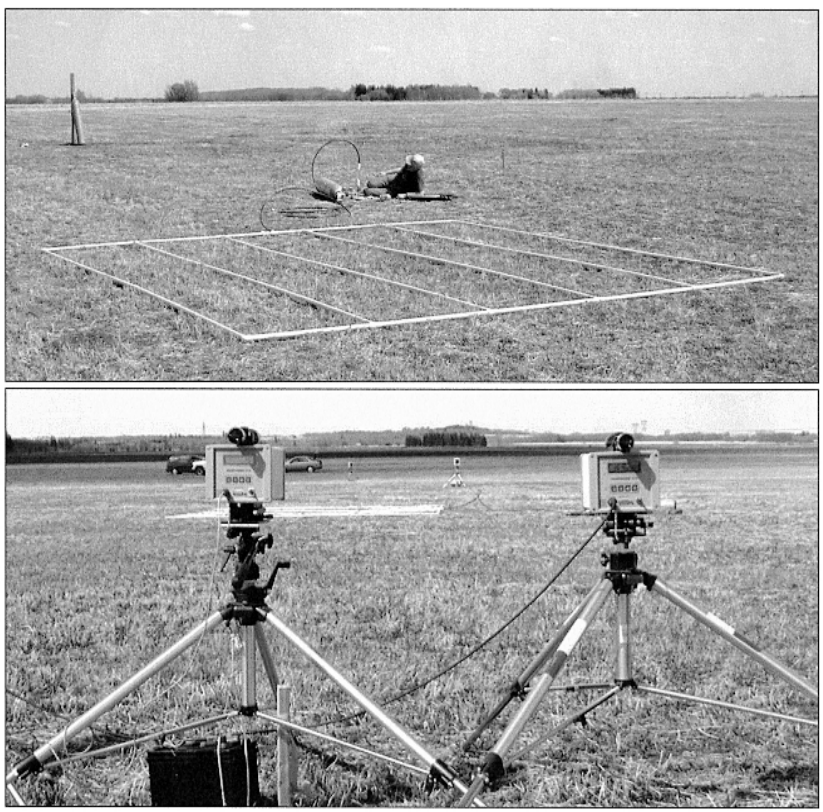
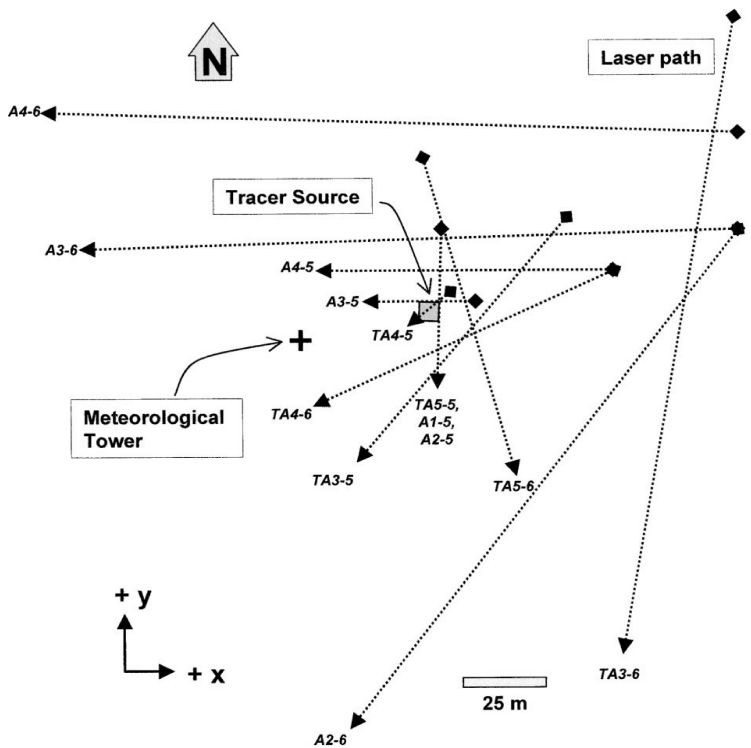


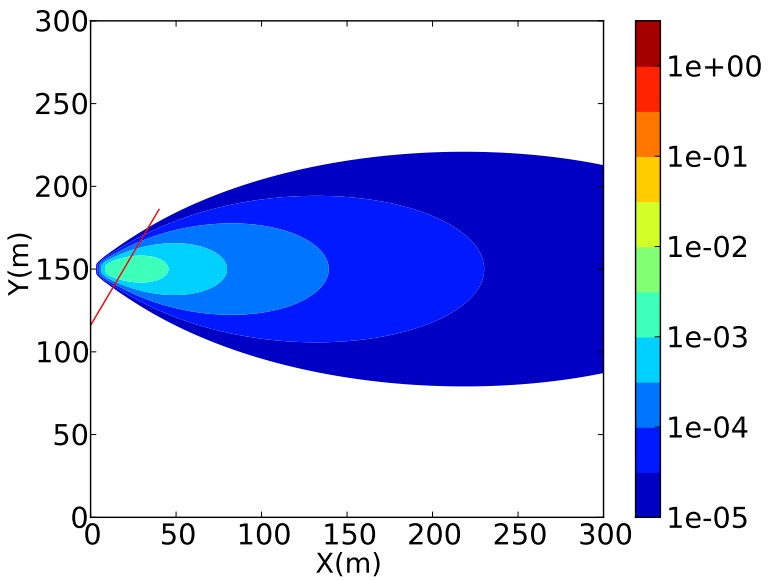
FIG. 2. The experimental site: (top) the synthetic tracer source and (bottom) two laser receivers in the foreground with retroreflectors in the background (the tracer source is also visible).



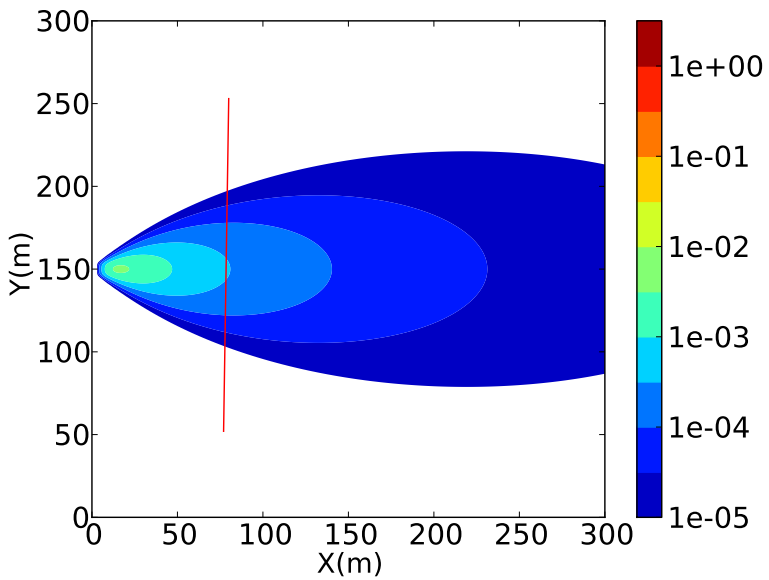
Flesch et al (2004) experiment

distance from source	stability	time	run
$16-23$	US	$14:45-16:30$	$TA3-5$
$75-81$	US	$14:45-16:30$	$TA3-6$
$11-18$	US	$15:45-17:30$	$TA5-6$
$15-44$	US	$16:15-18:00$	$TA4-6$
$58-98$	S	$04:30-07:00$	$A4-6$
$11-20$	S	$04:30-07:00$	$A4-5$
$23-34$	S	$05:15-07:00$	$A3-6$
$61-64$	$S-US$	$05:15-08:00$	$A2-6$

comparison with Flesch et al (2004): calc. concentrations (g/m^3)

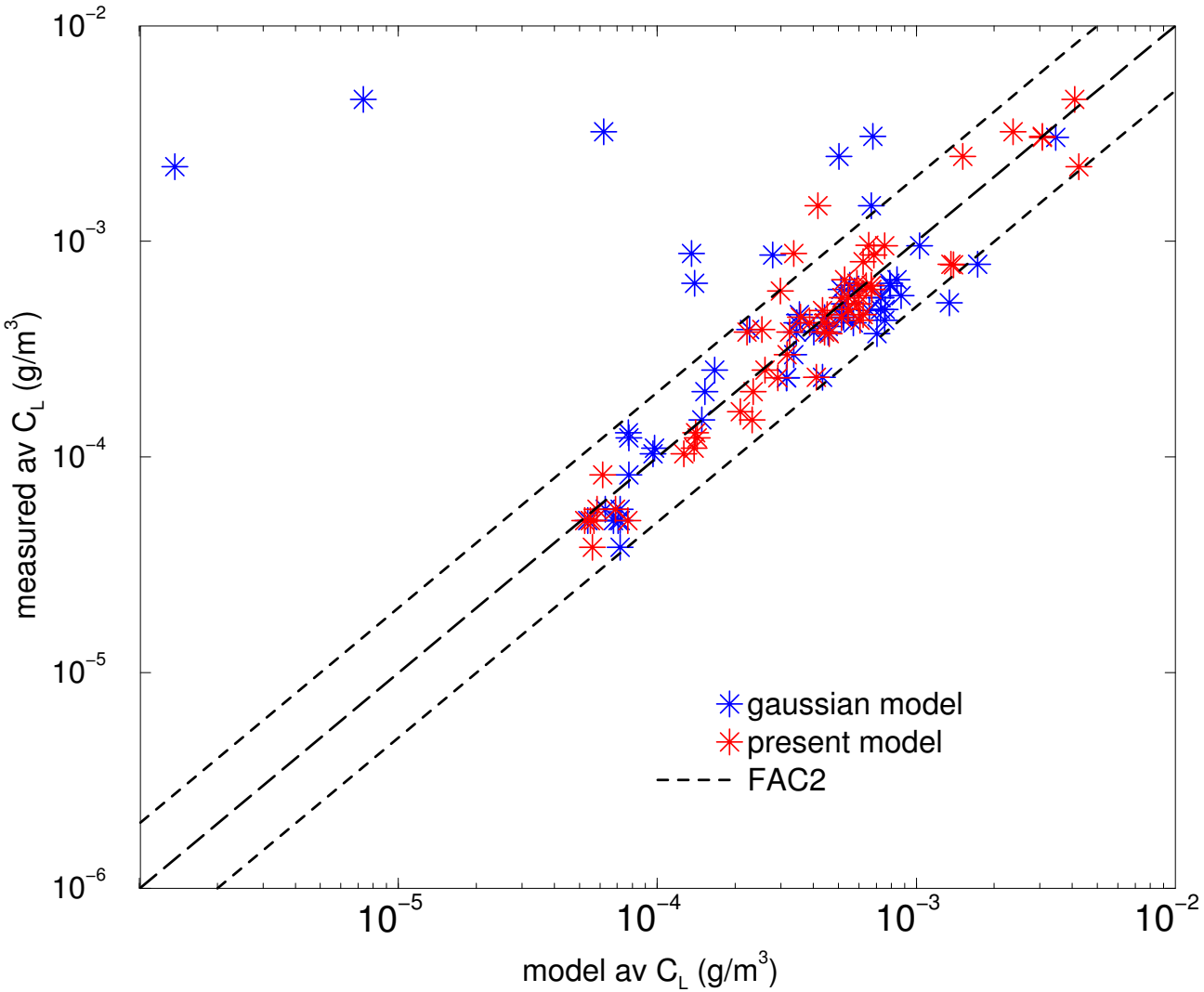


(a) ta3-5 16:00; from profile



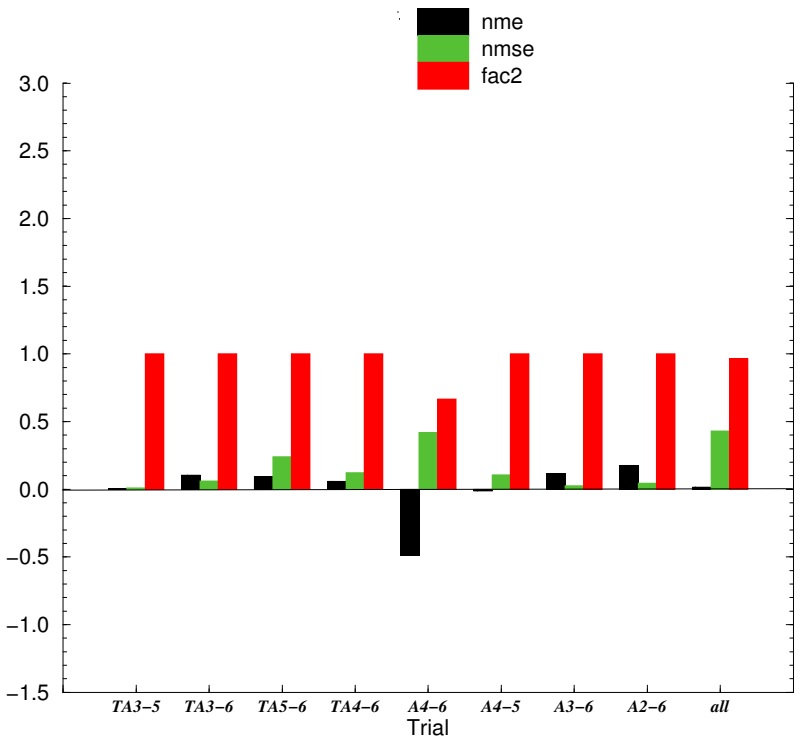
(b) ta3-6 16:00; from profile

Flesch et al (2004) data Vs model & Gaussian model

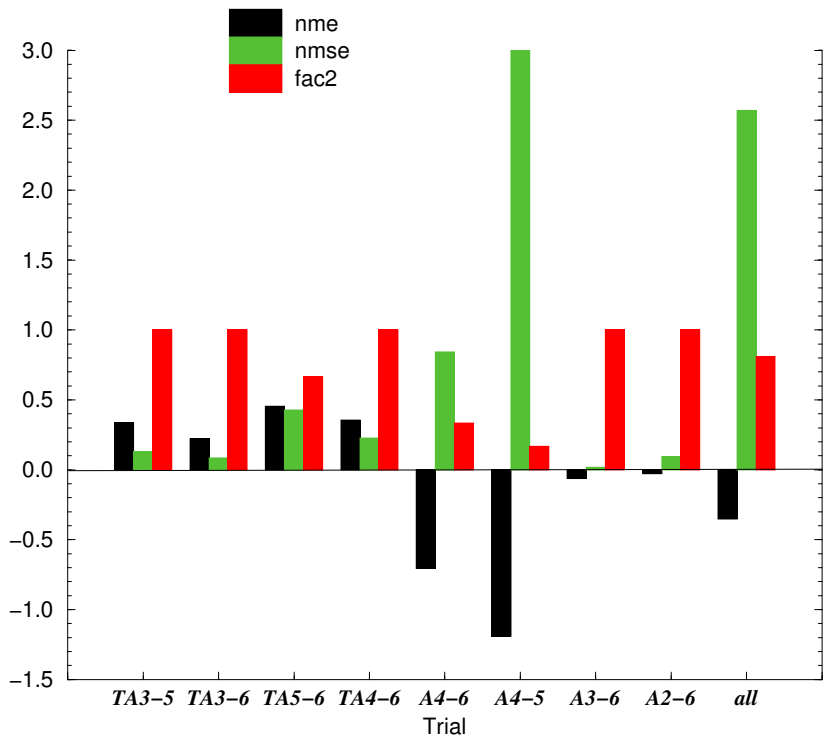


performance comparison, Chang & Hanna (2004)

$$-0.3 \leq NME \leq 0.3$$
$$NMSE \leq 2 - 3$$
$$FAC2 \geq 0.5$$



(a) model



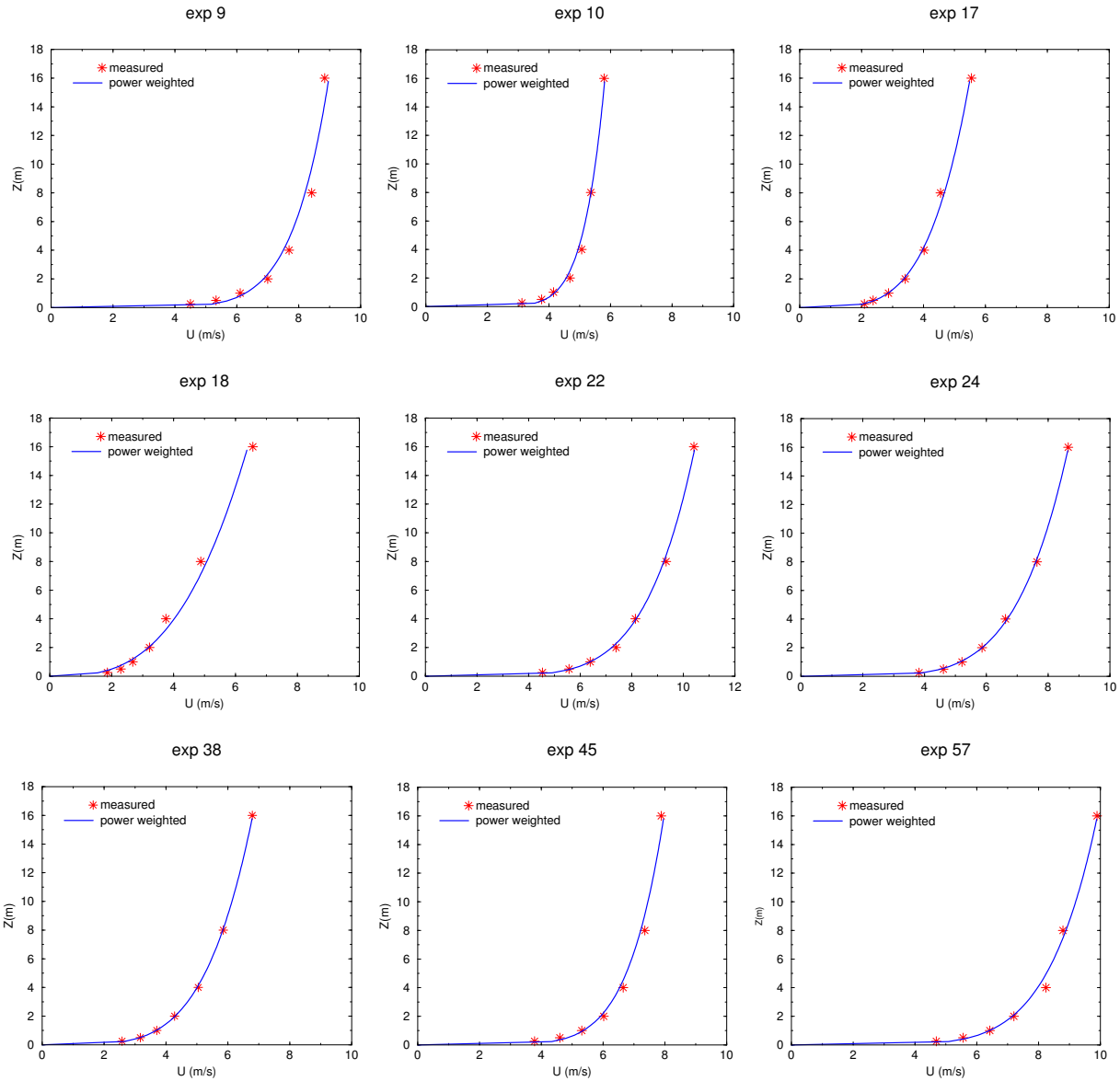
(b) gaussian model

Prairie Grass experiment

Run	*time*	*speed*(m/s)	*Temp* (^0C)	*dir*	dir_{opt}	u_* (m/s)	L (m)
9	10 : 05	7.00	27.39	199	201	0.49	-39
10	12 : 05	4.68	30.45	200	225	0.36	-12
17	20 : 05	3.41	27.44	172	179	0.32	70
18	22 : 05	3.22	23.52	188	184	0.40	47
22	00 : 05	7.39	26.42	170	173	0.53	276
24	23 : 05	5.86	21.89	146	143	0.44	307
38	05 : 05	4.28	19.82	161	170	0.37	151
45	17 : 05	6.02	35.04	153	164	0.41	-119
57	17 : 35	7.20	34.11	185	199	0.52	-248

U & T at 2m; u_* & L from $Z_1 = 0.5m, \quad Z_2 = 16m$

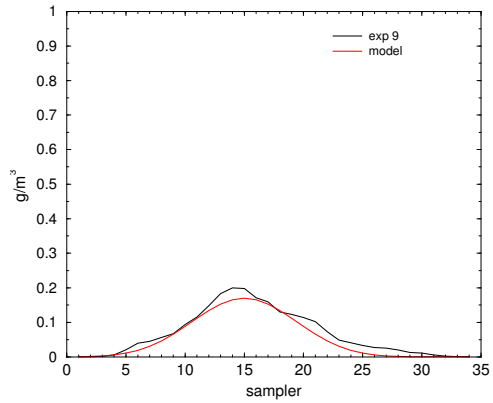
Measured & power law fitted velocity profiles



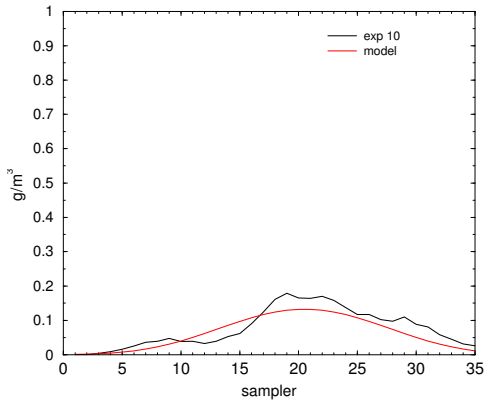
Input for the Gaussian model

run	$L(m)$	*time*	$U_{10}(m/s)$	stability class
9	-39	10:05	8.52	*D*
10	-12	12:05	5.47	*C*
17	70	20:05	4.80	*E*
18	47	22:05	5.30	*D*
22	276	00:05	9.60	*D*
24	307	23:05	7.89	*D*
38	151	05:05	6.09	*D*
45	-119	17:05	7.48	*D*
57	-248	17:35	9.06	*D*

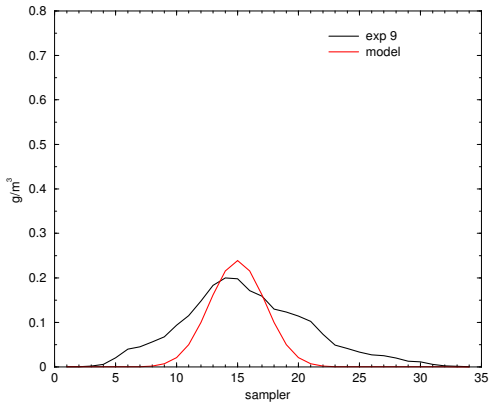
Measured & computed concentrations; R=50m; unstable



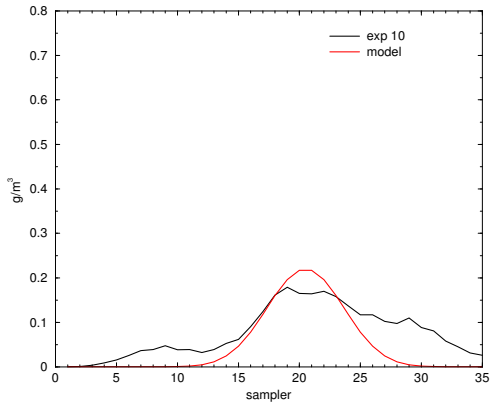
(a) exp 9; model



(b) exp 10; model

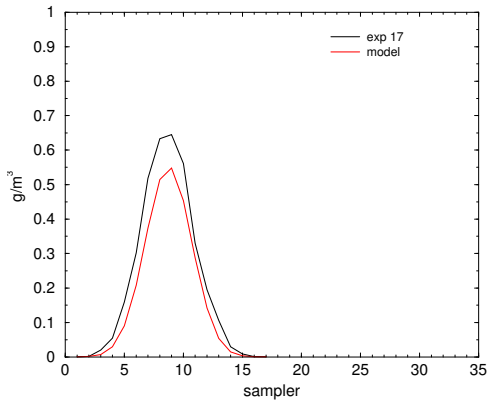


(c) exp 9; gauss

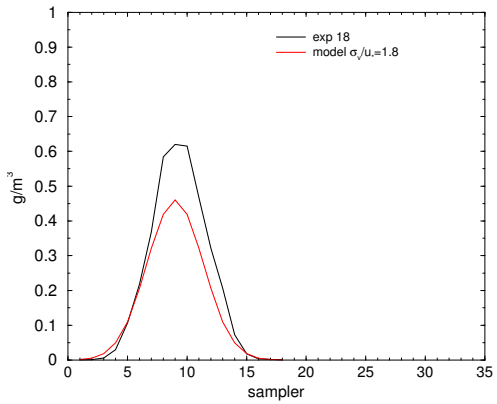


(d) exp 10; gauss

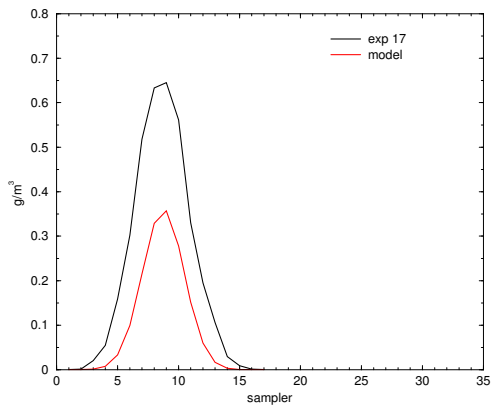
Measured & computed concentrations; R=50m; stable; $\sigma_v/u_* = 1.8$



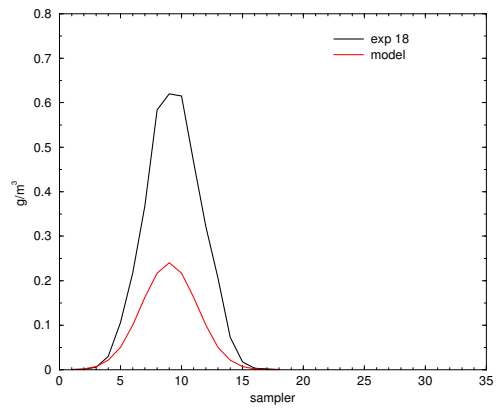
(a) exp 17; model



(b) exp 18; model

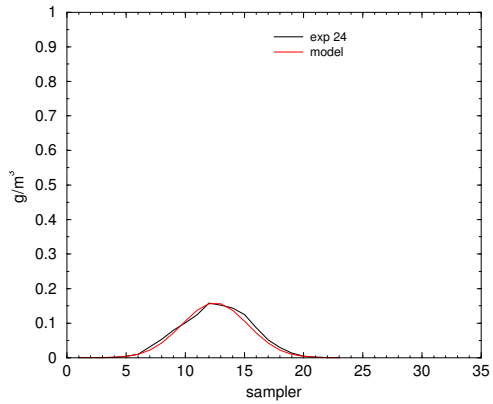


(c) exp 17; gauss

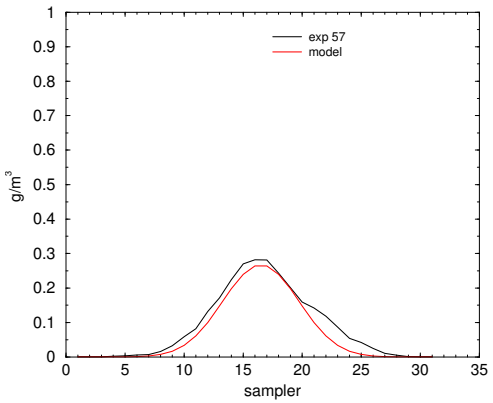


(d) exp 18; gauss

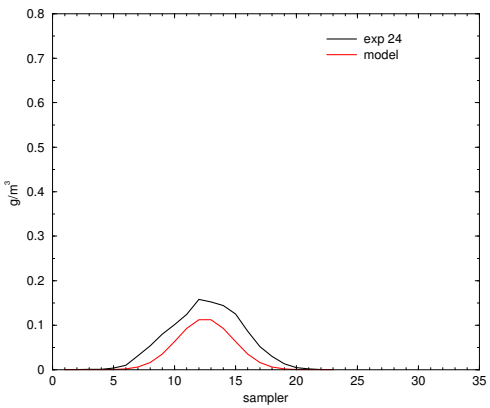
Measured & computed concentrations; R=50m; neutral; $\sigma_v/u_* = 2.2$



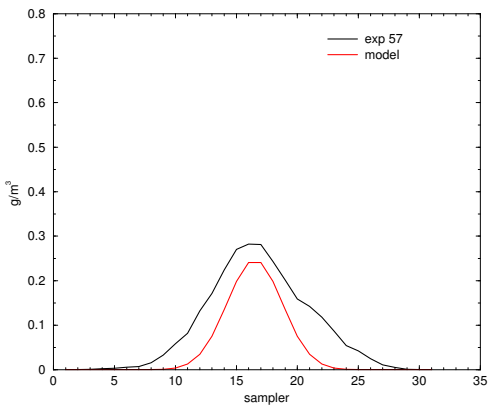
(a) exp 24; model



(b) exp 57; model

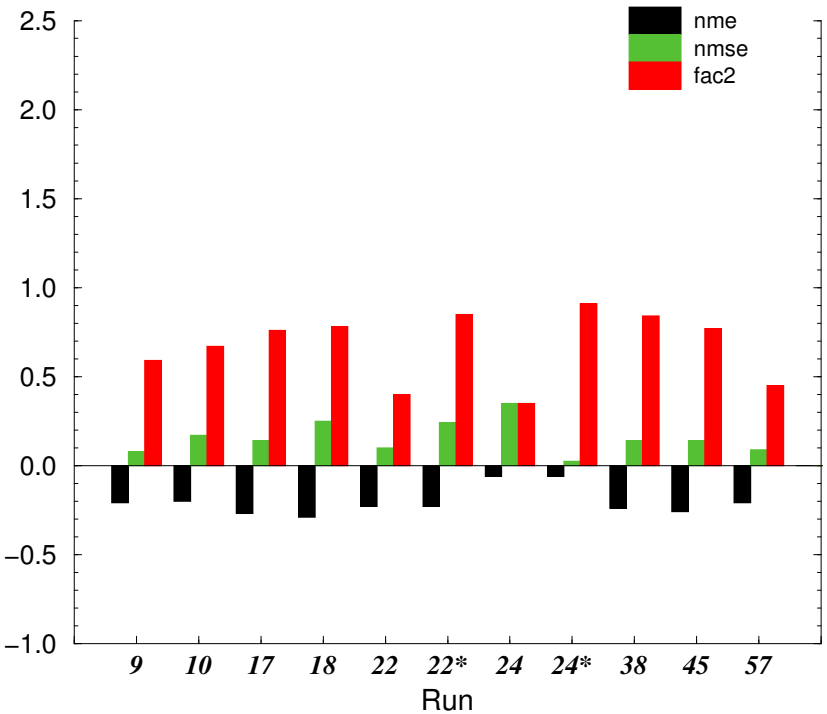


(c) exp 24; gauss

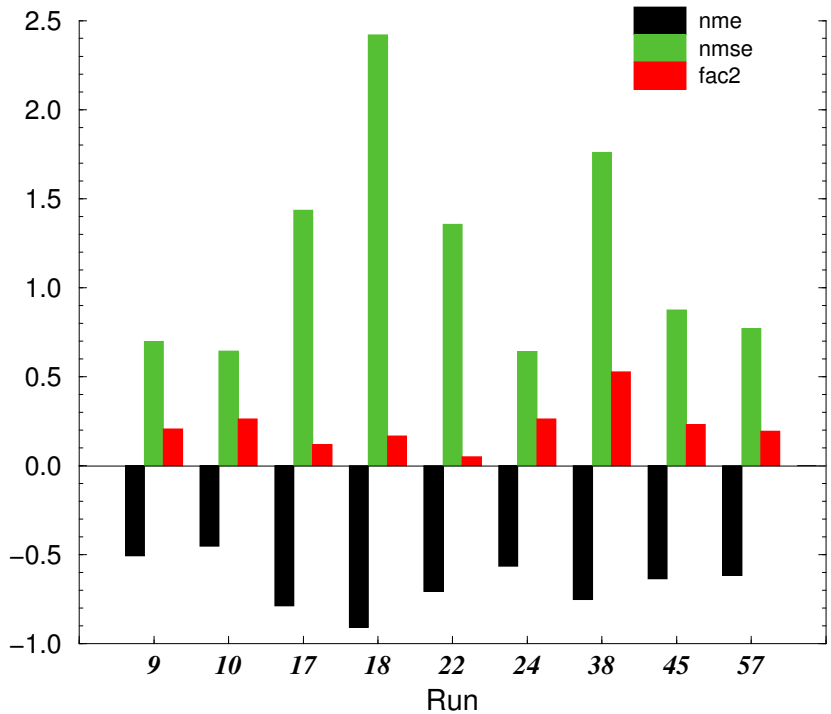


(d) exp 57; gauss

performance comparison with a Gaussian model



(a) model



(b) gaussian model

- NME & FAC2 criterion not satisfied by the Gaussian model
- both models - negative bias; Gaussian - $>$ three times larger

summary & conclusions

- The model c.w.i.c. were tested against similarity solutions and compared to a Gaussian model - test for power law.
  1. Stable and neutral conditions:
     Excellent agreement was achieved between the model & similarity concentrations in the first 200 meters.
     Significant disagreement between the Gaussian model & similarity.
  2. Unstable conditions:
     Good agreement in the first 200m. In the first 100 meter the model was superior to the Gaussian model. For larger distances the two models performed equally.
     The reasons for the deviations from similarity
     i. Near the source: the difficulty to represent velocity profile in unstable conditions near the ground as power law.
     ii. Far from the source: the rapid growth of the cloud height requires a different set of coefficients ($H > 1$).

summary & conclusions

- The model was tested against two field experiments and compared to a Gaussian model:
  i. Prairie Grass (R=50m)
  ii. Flesch et. al. (2004) (average along line of sight)
- The model performed well even in unstable conditions.
- The performance of the model was superior to that of the Gaussian model
  1. Much smaller relative bias.
  2. Better representation of the cloud width.

Acknowledgement

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