An analytical urban dispersion model compared with four field experiments

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Background

Upper bound (Robins and Cheng 2003)

$$\frac{CU}{Q} = \frac{K_D}{x^2}$$

- Decay of concentration $C \propto x^{-2}$ (Venkatram et al. 2005)
- Constant K_D depends on stability (Hanna et al. 2007)

Gaussian plume model

Mean concentration field

$$c = \frac{Q}{\pi U \sigma_y \sigma_z} \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

Approximations

- Uniform wind profile U
- Boundary accounted for by reflections

Vertical spread (Hunt and Weber 1979)

Neutral atmosphere, ground-level release

Statistical diffusion theory

$$\frac{d\sigma_z^2}{dt} = 2\int_0^t \langle w(t)w(t')\rangle\, dt' \qquad \Longrightarrow \qquad \sigma_z^2 = b^2\sigma_w^2 t^2$$

Assumptions

- $\sigma_w \propto u_*$
- $\langle w \rangle \propto u_*$
- $T_L \propto L_w/\sigma_w$
- $L_w \propto \langle z \rangle$; $\qquad \langle z \rangle = \langle w \rangle\, t \qquad \Longrightarrow \qquad T_L \propto t$

Mixing height effect

Far-field limit

$$\lim_{x\to\infty} c = \frac{Q}{\sqrt{2\pi}U\sigma_y L_z}\exp\left(-\frac{y^2}{2\sigma_y^2}\right)$$

$$\lim_{x\to\infty} \sigma_z^2 = \frac{2L_z^2}{\pi}$$

Bounded equation

$$\sigma_z^2 = \frac{b^2\sigma_w^2 t^2}{1 + b^2\sigma_w^2 t^2 \pi/(2L_z^2)}$$

Horizontal spread (Taylor 1921)

Homogeneous stationary turbulence

Statistical diffusion theory

$$\sigma_y^2 = 2L_y^2 \left[\frac{t}{T_y} + \exp\left(-\frac{t}{T_y} \right) - 1 \right]$$

Asymptotic limits

$$\sigma_y^2 = \sigma_v^2 t^2 \qquad \text{for } x \ll UT_y$$

$$\sigma_y^2 = 2\sigma_v^2 T_y t \qquad \text{for } x \gg UT_y$$

Decay slope

Range	Slope σ_y	Slope σ_z	Slope C
$x \ll UT_z$	x	x	x^{-2}
$x \gg UT_y$	$x^{1/2}$	x^0	$x^{-1/2}$

Asymptotic equations

$$C = \left(\frac{Q}{L_y L_z} \frac{UT_y T_z}{\pi b} \right) x^{-2} \qquad \text{for } x \ll UT_z$$

$$C = \left(\frac{Q}{L_y L_z} \sqrt{\frac{T_y}{4\pi U}} \right) x^{-1/2} \qquad \text{for } x \gg UT_y$$

Data analysis

Scaled C

- $C \quad \Longrightarrow \quad \frac{CUL_yL_z}{Q}$
- L_y, L_z Boundary-layer turbulence lengths
- $U = U(H)$

Scaled x

- $x \quad \Longrightarrow \quad \frac{x}{UT}$
- $T = \sqrt{T_yT_z} \qquad T_y = L_y/\sigma_v; \qquad T_z = L_z/\sigma_w$
- In-canopy σ_v and σ_w from data, or
- $\sigma_v = 0.5u_c$ and $\sigma_w = 0.33u_c$ (Hanna et al. 2007)

Back of the envelope

Asymptotic C

	$x \ll UT$	$x \gg UT$	
$\frac{CUL_yL_z}{Q} =$	$0.64\left(\frac{x}{UT}\right)^{-2}$	$0.38\left(\frac{x}{UT}\right)^{-1/2}$	Stable
	$0.32\left(\frac{x}{UT}\right)^{-2}$	$0.32\left(\frac{x}{UT}\right)^{-1/2}$	Neutral

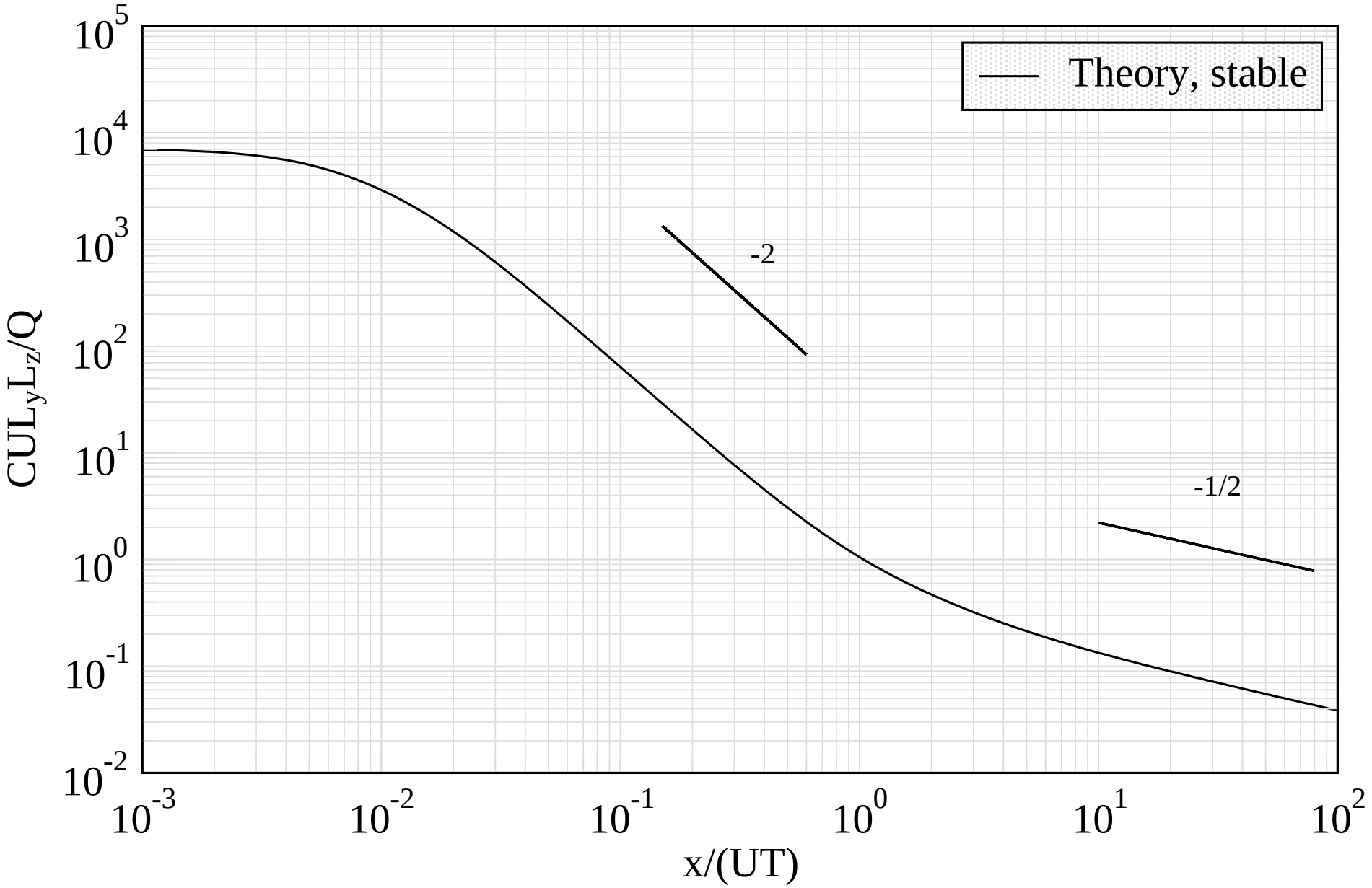
Datasets

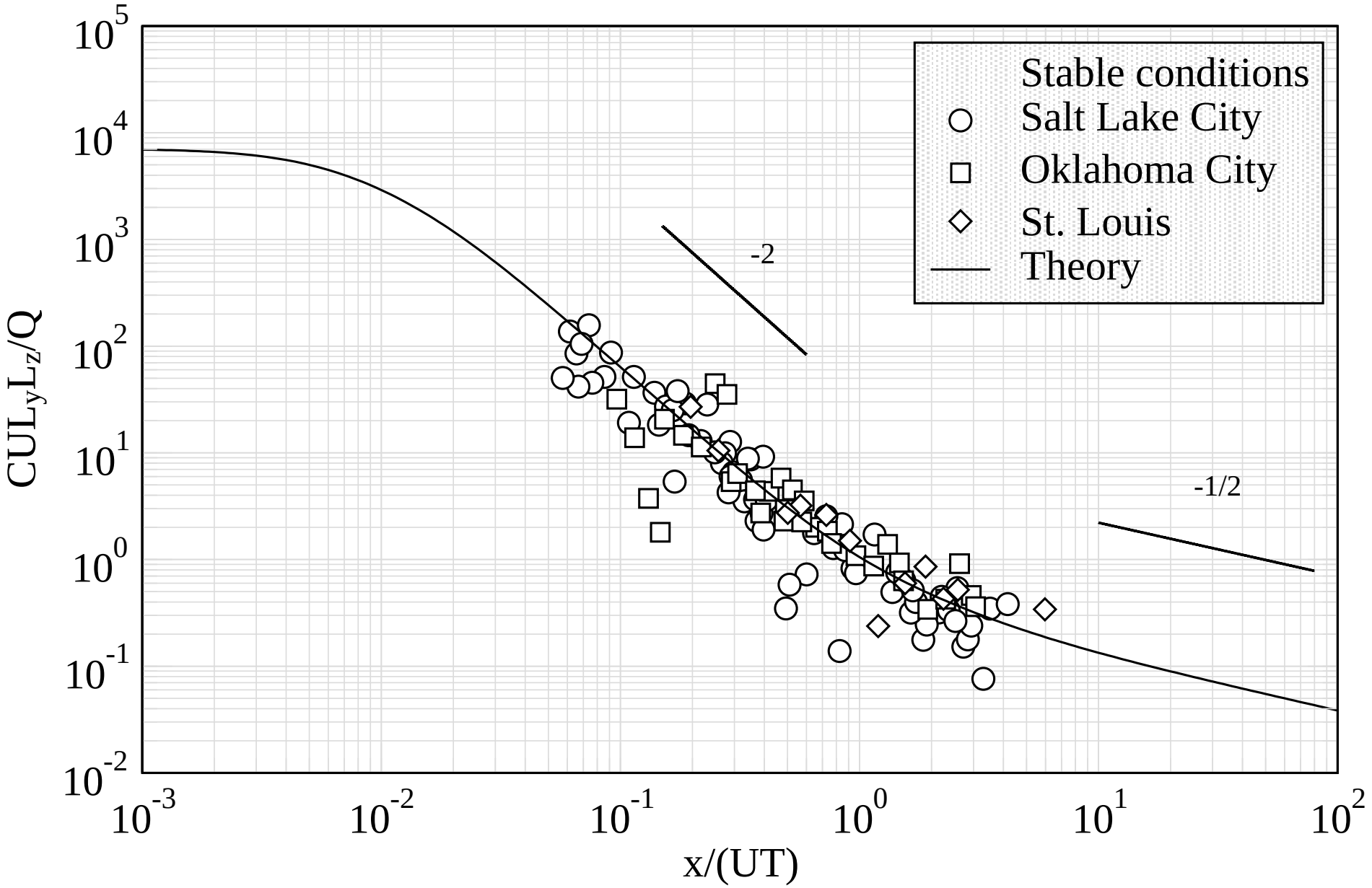
- Salt Lake City (URBAN 2000) (Hanna et al. 2003)
- Oklahoma City (JU2003) (Hanna et al. 2007)
- St. Louis (McElroy and Pooler 1968)
- London (DAPPLE) (Neophytou and Britter 2004)

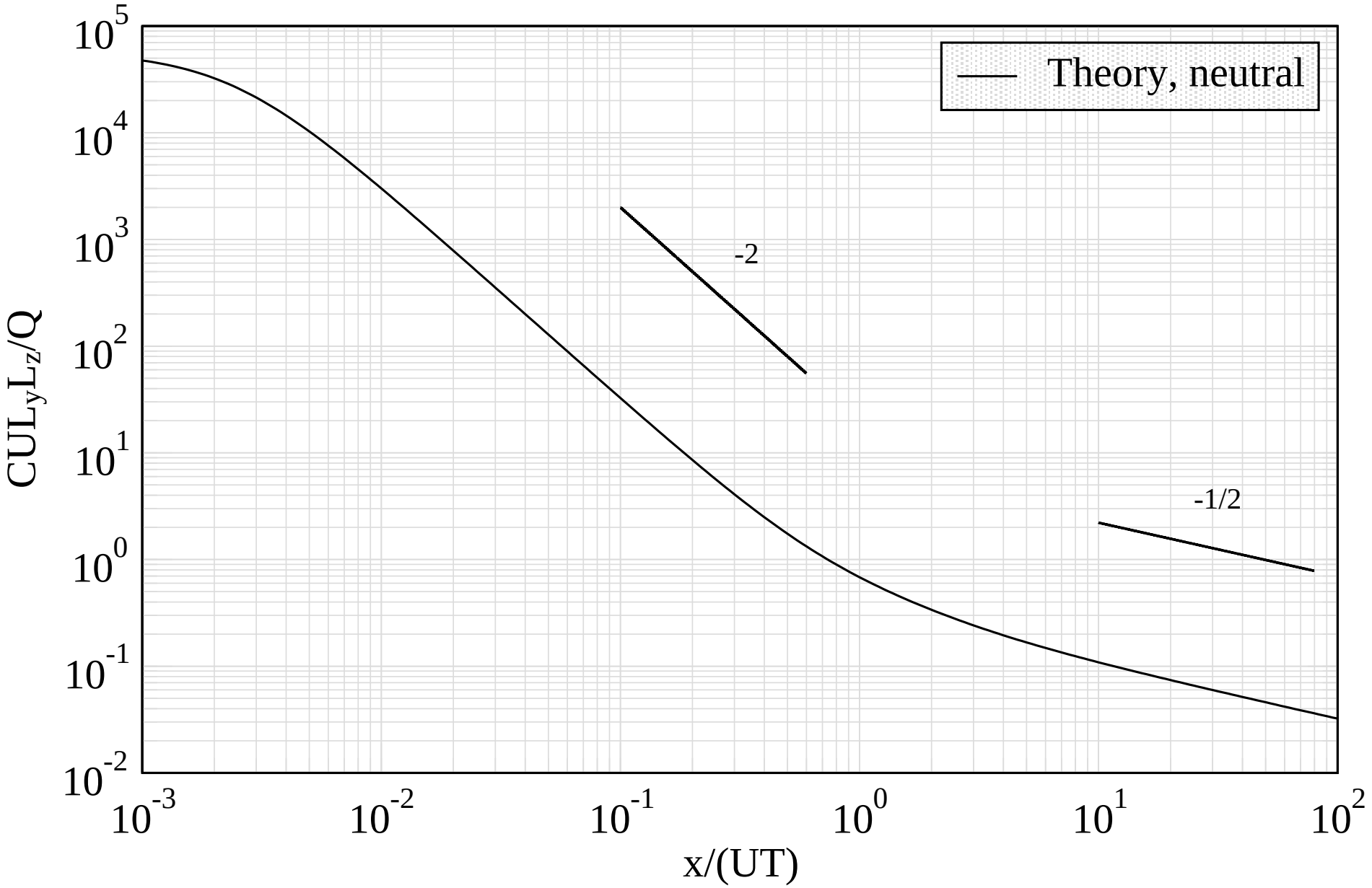
Summary of turbulence and flow

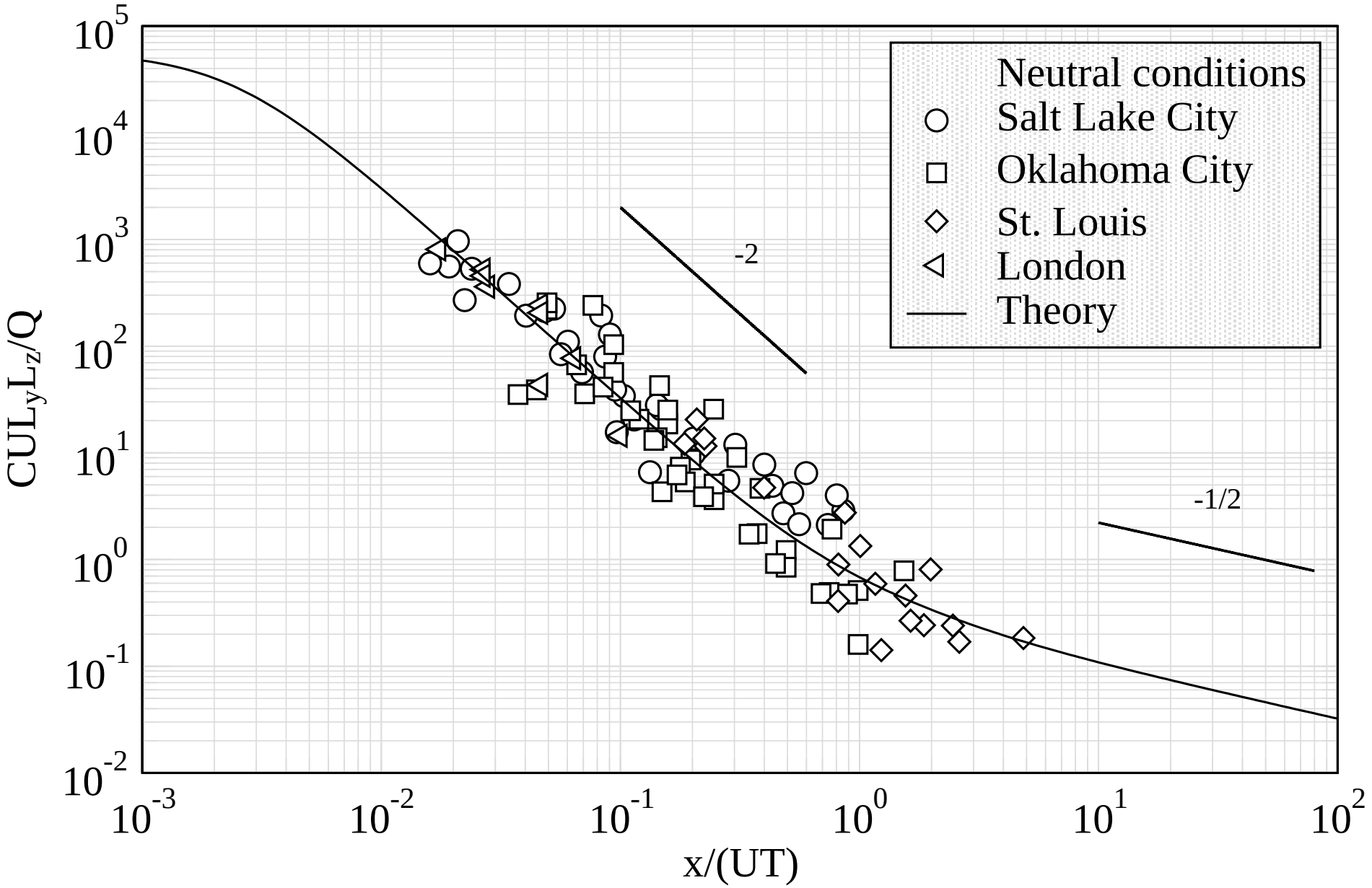
	L_y [m]	L_z [m]
Stable	1000	200
Neutral	2000	800

Experiment	Range [m]	Atm.	σ_v [ms^{-1}]	σ_w [ms^{-1}]	T_y [s]	T_z [s]
Salt Lake City	150-6000	Stab	0.25	0.16	4082	1237
		Neut	0.52	0.34	3883	2354
Oklahoma City	200-4000	Stab	0.99	0.68	1010	294
		Neut	1.09	0.70	1835	1143
St. Louis	600-16000	Stab	0.45	0.30	2208	669
		Neut	4.05	2.77	494	289
London	75-430	Neut	1.08	0.72	1845	1118


Theory, stable
-2
-1/2
CUL_yL_z/Q
x/(UT)
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}


Stable conditions
Salt Lake City
Oklahoma City
St. Louis
Theory
-2
-1/2
CUL_yL_z/Q
x/(UT)
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
10^2


Theory, neutral
-2
-1/2
CUL_yL_z/Q
x/(UT)
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
10^2


Neutral conditions
Salt Lake City
Oklahoma City
St. Louis
London
Theory
-2
-1/2
CUL_yL_z/Q
x/(UT)
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}