Assignment # 1

1) Integrate the linear first order constant wind speed advection equation for the following cases;

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

a) Case #1;

- Initially condition, $u(x,t) = u(x,0) = \sin 2\pi x, \quad 0 \le x \le 1$
- c = 1
- $\Delta t = 1/64$
- The domain is divided into 16 intervals
- Thus the CFL condition, $\mu = c\frac{\Delta t}{\Delta x} = 0.25$

Verify that the solution propagates eastward with a period of about 64 time steps, and determine whether the amplitude exceeds 1.

b) Case #2;

Repeat the above experiment for μ = 0.5, 1.0, and 2.0 (c and Δx are fixed).

Verify that the solution "blows up" for $\mu > 1$, and compare the results of the above three stable integrations (hint: adjust frequency of output to facilitate comparison between solutions, e.g., phase speed comparison).

c) Case # 3;

With the same initial condition and $\mu = 0.8$, run the model with 32, 16, and 8 intervals (hint: again adjust frequency of output). Then, comment on the phase speed and amplitude differences (if any) between the three runs.

d) Case # 4;

Also run the following two cases using 16 intervals and $\mu = 0.8$ for the following initial conditions

i) $u(x,0) = \frac{1}{2}(\cos 2\pi x + \cos 4\pi x)$

ii) $u(x,0) = \frac{1}{2}(\cos 2\pi x + \cos 12\pi x)$

Is there computational dispersion? If so, is it worse with the smaller wave present?

Note: In all cases above, use centered differencing in space and time (except initially use a forward time-step). For all experiments, the initial condition and the solution will be defined on a cyclic domain $0 \le x \le 1$. Thus, u(0)=u(1). Be sure that your solution satisfies this condition at all times.