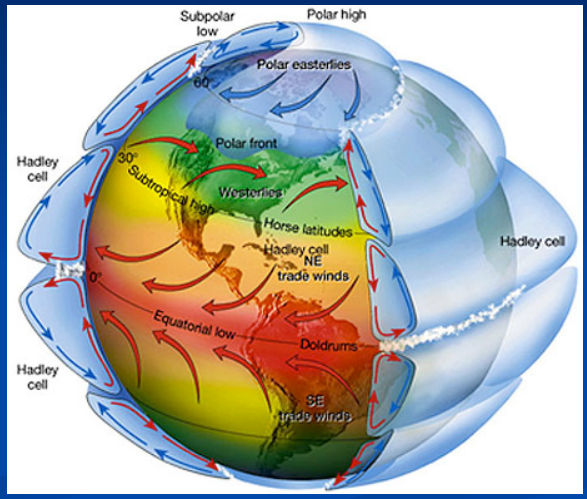
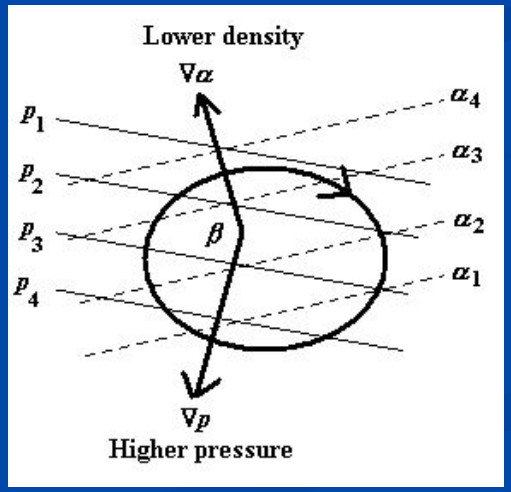
CLIM-111/PHYS-111
Introduction to Atmospheric Physics and Chemistry
Lecture Notes 7: Atmospheric Dynamics

1. **Kinematics of Large-Scale Horizontal Flow**
2. **Dynamics of Horizontal Flow**
3. Primitive Equations
4. The Atmospheric General Circulation

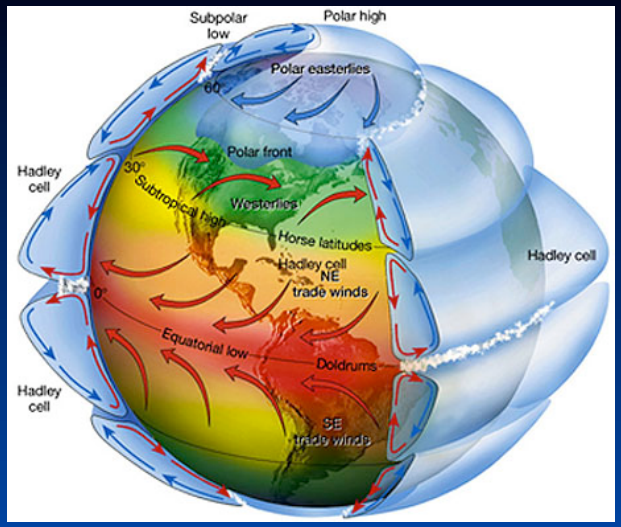








Atmospheric Dynamics: The Basics



1) Real Forces:

Pressure gradient	$-\partial P/\partial x$
Friction	$-\nu\,\partial^2 U/\partial z^2$
Gravity	$-\rho\,\mathbf{g}$

2) Reference Frames:

Inertial (non-accelerating)	Newtonian Dynamics
Non-Inertial (accelerating)	Apparent Forces (**Coriolis**)

3) Time Tendency:

Fixed observer	Material Deriviative
Observer "riding" motion	Lagrangian Deriviative

4) Conservation Laws:

Mass	m
Momentum (force equation)	linear + angular
Energy	$1/2mv^2 + \rho C_p T + \rho gz$

5) Scaling of the Equations of Motion:

Geostrophic Balance	Rossby Number
Turbulence	Reynold's Number

Dynamics of Horizontal Flow

- According to Newton's 1st law of motion, a body will change its velocity only if acted upon by an unbalanced force.
- Newton's 2nd law states that the acceleration (A) is directly proportional to the magnitude of the unbalanced (or net force) and inversely proportional to the mass (M) of the body.

$$A = \sum F / M$$

- Where $\sum F$ is the net force (so that the sum of the all forces).

Horizontal Pressure-Gradients & Wind

- So, to determine the state of motion of a body, one must identify all of the forces that act on the body
- In case of atmosphere, the following are the most significant forces (per unit mass) that affect air motion
  - Pressure-gradient (PG) force
  - Gravity (G) force
  - Coriolis (C) force
  - Friction (F) force
- Thus, for atmosphere, the equation governing atmospheric motion may be written

$$A = PG + G + C + F$$

Reference Frame

In an inertial (non-accelerating) reference frame Newton's Laws of Motion can be directly applied to a parcel of gas in order to determine its time tendency (acceleration).

Euler's Equation:

$$\frac{dV}{dt} = PG + G + C + F$$

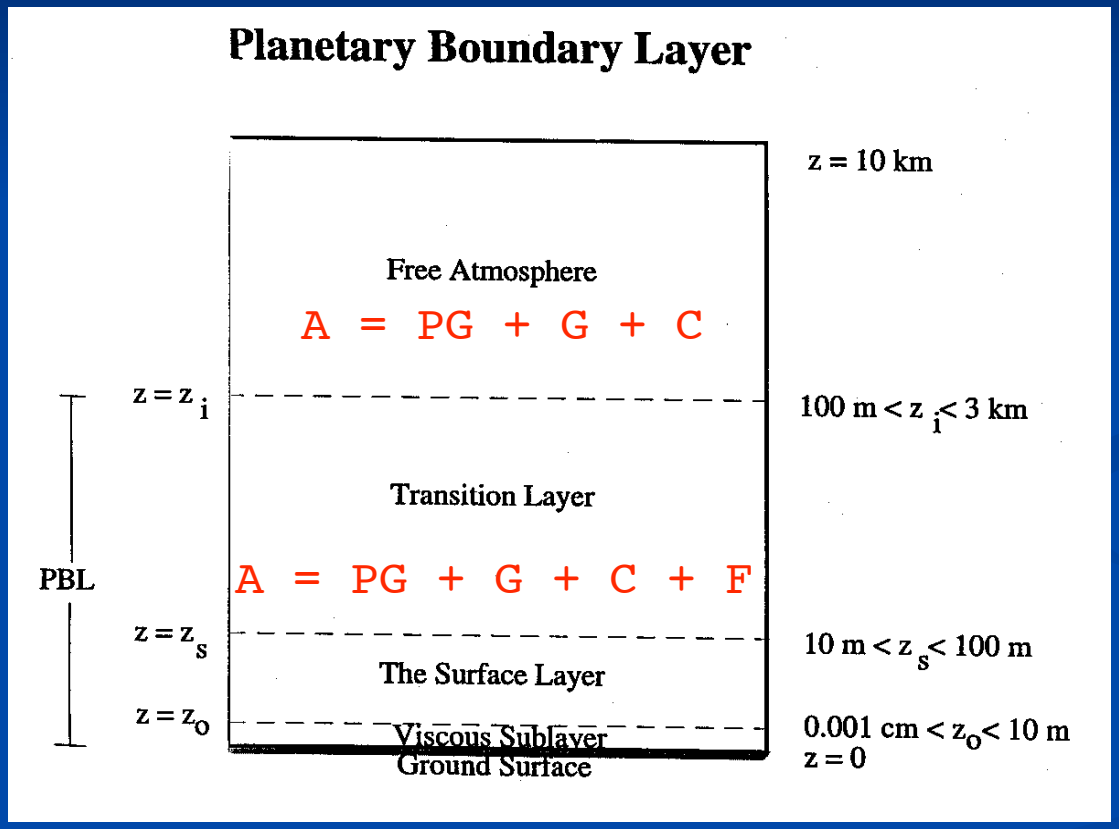
Dv/Dt is the material or **advective deriviative** in an inertial reference frame. It can be related to the Lagrangian deriviative ("riding" the parcel) via

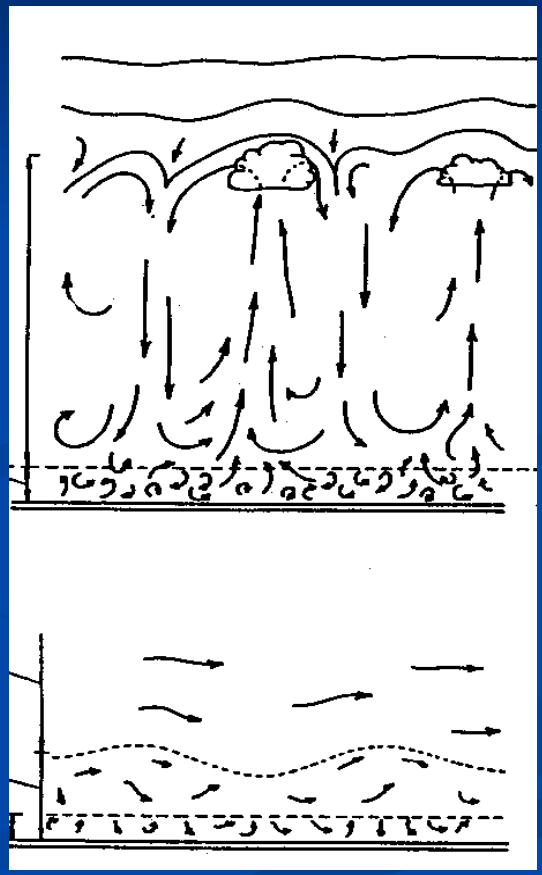
$$\frac{dV}{dt} = \frac{\partial V}{\partial t} + u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + w\frac{\partial V}{\partial z} = PG + G + C + F$$

Horizontal Pressure-Gradients & Wind

- Of the four forces in the above equation, **PG** and **G** forces can be thought of as **the basic forces**, since without them there would be no motion.
- They are the "driving" forces and are independent of the velocity, i.e., they exist regardless of the state of the motion of the air.
- In contrast, **C** and **F** forces are dependent on the velocity, i.e., they arise only when air is moving.
- So, these forces **help sustain** and even **enhance** the development of the Earth's pressure systems.

Friction Layer (PBL)





The Pressure Gradient Force

The vertical force balance is known as hydrostatic equilibrium:

The vertical force per unit mass:

$$\frac{1}{\rho}\frac{dp}{dz} = -g$$

In general the total pressure force:

$$P = -\frac{\nabla p}{\rho}$$

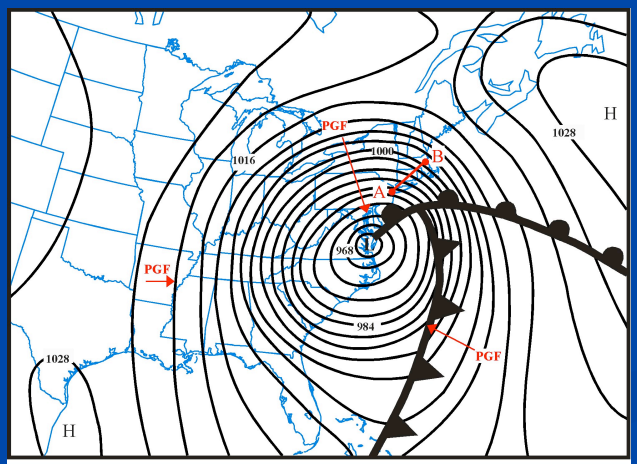
Horizontal components of the pressure gradient force:

$$P_x = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad P_y = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

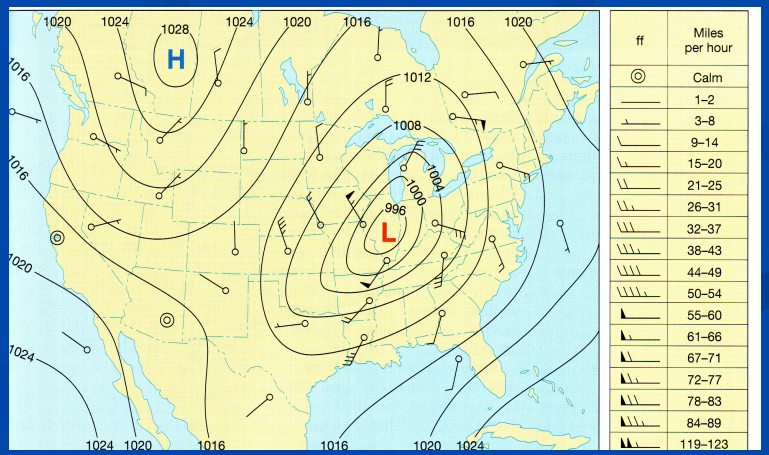
Pressure-Gradient Force (PG)

$$P_x = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad P_y = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

- ✓ Analogous to the slope of a hill,
- ✓ Closely spaced isobars indicate a steep PG (steep hill) and strong winds.
- ✓ Widely spaced isobars indicate a weak PG (gentle hill) and light winds.
- ✓ Acceleration is thus downslope (i.e., from high to low pressure).

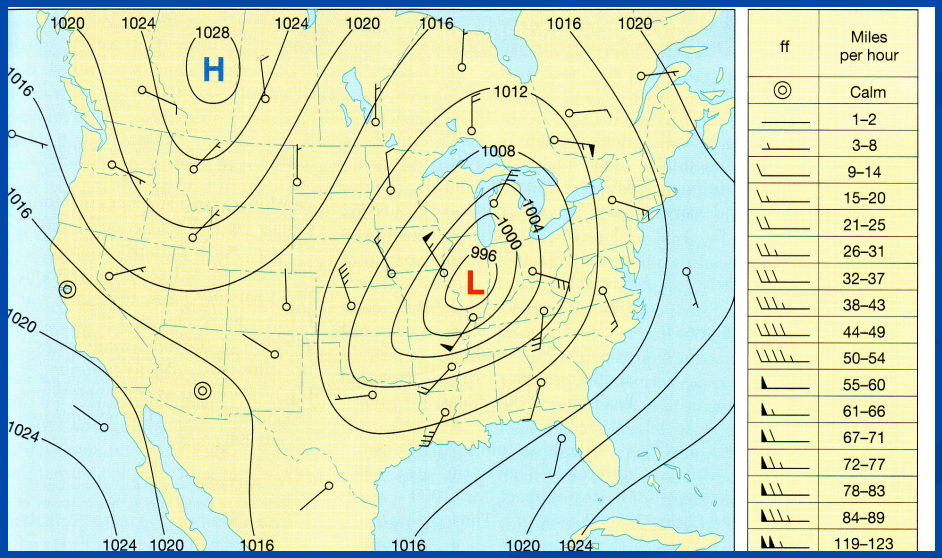


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Horizontal Pressure-Gradients & Wind

- If the **PG** force was the only force acting on the wind, what happens?



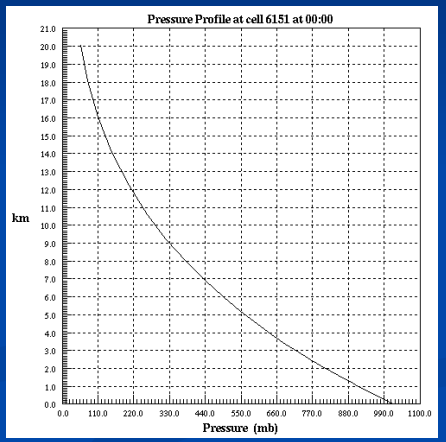
Horizontal Pressure-Gradients & Wind

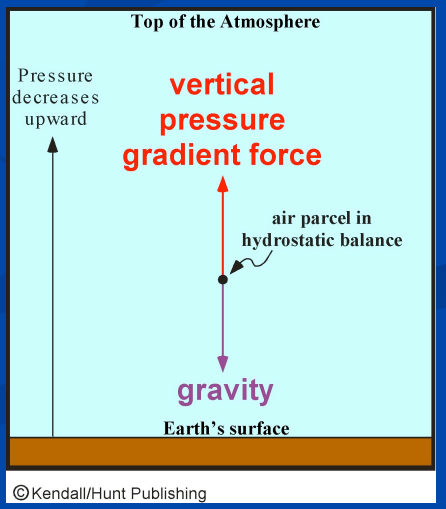
- If the PG force was the only force acting on the wind, air would flow from areas of higher pressures to areas of lower pressure and lows would be quickly filled.
- Thus, high and low pressure systems could never sustain themselves.
- Instead, Earth's atmosphere would develop only very weak and short-lived pressure systems.
- As a result, most locations would experience long periods of very stable air that would occasionally be disturbed by gentle breezes.
- However, weather is more varied.
- Then, there must be other forces affecting air motion (friction and Coriolis).

Vertical Pressure-Gradients

- We have learned that airflow is from **H** to **L** pressures.
- We also know that air pressure is highest near Earth's surface and exponentially decreases with height.
- Then, one wonders why air does not rapidly accelerate upward and **escape** into space.
- Because, **G** (which acts in the opposite direction of **PG**) balances **PG**.
- The balance between these two forces is called **hydrostatic equilibrium or balance**.

$$\frac{1}{\rho}\frac{dp}{dz} = -g$$



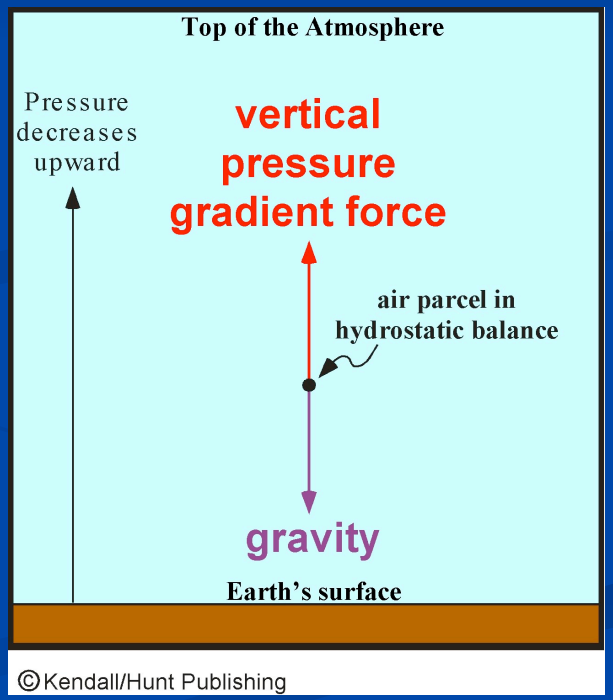


Vertical Pressure-Gradients

- Unlike **PG** which can act in **any direction**, the gravitational pull of the Earth is always directed **downward**, toward the center of the earth.
- The strength is proportional to the **mass** of the air parcel.
- Gravity **varies slightly** with latitude and altitude and even with the irregular mass distribution.
- For most atmospheric purposes, the following mean sea level value may be used

$$G = 9.806\ m/s^2$$

$$\frac{1}{\rho}\frac{dp}{dz} = -g$$

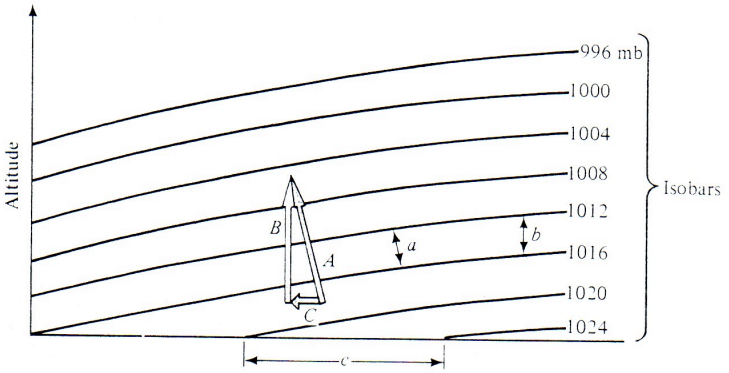


Vertical Pressure-Gradients

- Typical values for the distances **b = 33m** and **c = 300km**.
- Thus, the slope would be only **33/300,000** or about **1km** height change in almost **10,000km**
- The horizontal pressure gradient in this example would be **4mb/300km** (**$0.13X10^{-4}$ mb/m**),
- While the vertical pressure gradient would be **4mb/33m** (**0.12 mb/m**)
- The pressure gradient force in the vertical is thus almost **10,000** times that in the horizontal.

$$\frac{1}{\rho}\frac{dp}{dz} = -g$$

$$P_x = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$



a = perpendicular distance between isobars
b = vertical distance between isobars (exaggerated here)
c = horizontal distance between isobars

A = total or "resultant" pressure gradient
B = pressure gradient (component) in the vertical
C = pressure gradient (component) in the horizontal

Figure 4.2 ***Distribution of isobaric (equal-pressure) surfaces in a vertical cross section.***

Vertical Pressure-Gradients

- The horizontal and vertical components of the acceleration produced by the pressure gradient can be obtained by dividing by density.
- Using the standard sea level density value,

  $d = 1.230 \times 10^{-3}\ g/cm^3$
- The horizontal component of the acceleration due to the pressure gradient would be $0.11\ cm/s^2$
- While the vertical component would be $976\ cm/s^2$

$$\frac{1}{\rho}\frac{dp}{dz} = -g$$

$$P_x = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

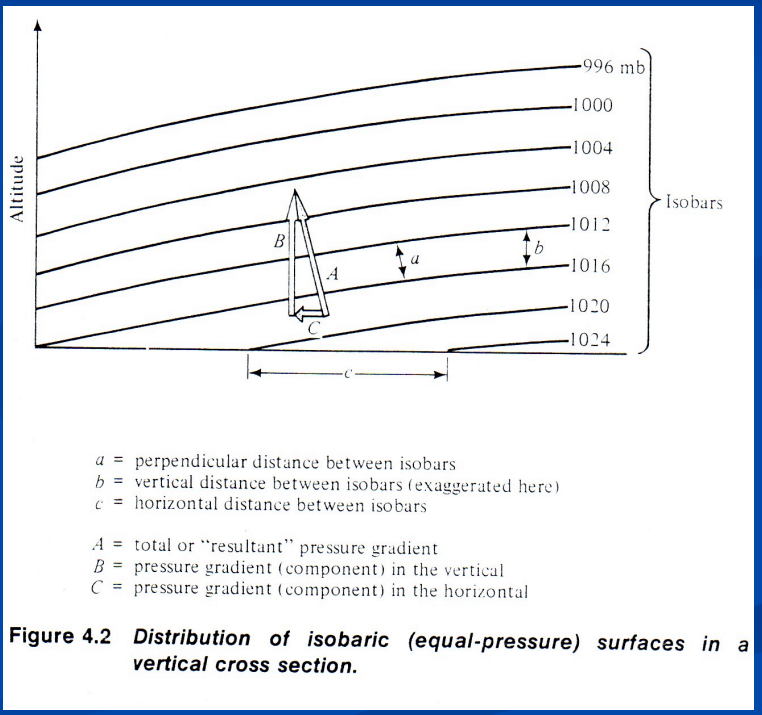
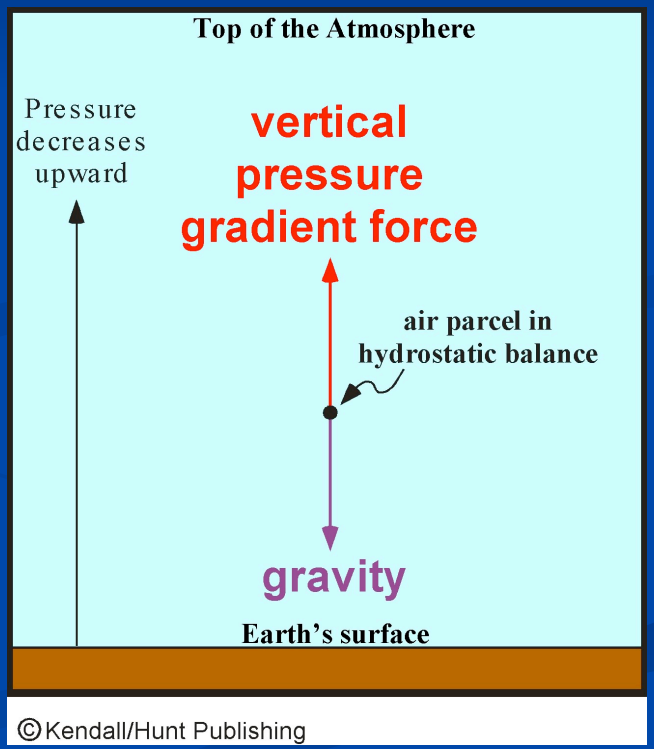


Figure 4.2 ***Distribution of isobaric (equal-pressure) surfaces in a vertical cross section.***

Vertical Pressure-Gradients

- So, these two relatively large forces (G & PG) that due to gravity (980 cm/s^2) and that due to the vertical pressure gradient (976 cm/s^2) forms the hydrostatic balance.
- It is the net difference between these oppositely directed vertical forces that determines whether a parcel will accelerate upward or downward and at what rate it will do it so.
- Since gravity is essentially constant, the vertical acceleration will depend on the magnitude of the upward component of the pressure gradient force.

$$\frac{1}{\rho}\frac{dp}{dz} = -g \quad \rightarrow \quad 976\ cm\,s^{-2} \cong 980\ cm\,s^{-2}$$



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The Horizontal Equations of Motion

$$\frac{dV}{dt} = PG + C + F$$

East-West direction (x, u positive toward East):

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + F_x$$

North-South direction (y, v positive toward North):

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + F_y$$

where $F_{x,y}$ is the external force (e.g. friction)

Geostrophy and the Rossby Number

E-W: $\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + F_x$ **N-S:** $\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + F_y$

Outside of the Boundary Layer (above ~1km altitude) the friction components are insignificant. If the acceleration terms are small, then those too may also be ignored. To determine if du/dt & dv/dt can be ignored, compare their magnitude to the Coriolis Force for a typical velocity (U), lengthscale (L) and timescale (T ~ L/U)

$dU/dt \sim U/T \sim U^2/L$;
$fu \sim f\,U$

The non-dimensional Rossby Number: $(dU/dt)/fu \sim U/fL$ is a measure of the relative magnitude of the acceleration to Coriolis terms.

$$R_o = U/fL$$

The Horizontal Equations of Motion

Above ~1km altitude (outside the boundary layer) $F_{x,y}=0$:

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = fv \qquad -\frac{1}{\rho}\frac{\partial p}{\partial y} = fu$$

Note that this force balance is diagnostic, not prognostic, i.e., there is no tendency or time evolution. This is called the Geostrophic Balance. The wind velocity that exactly solves these equations is known as the Geostrophic Wind.

$$u_g = -\frac{1}{f\rho}\frac{\partial p}{\partial y} \qquad v_g = \frac{1}{f\rho}\frac{\partial p}{\partial x}$$

Geostrophy

Above ~1km altitude (outside the boundary layer) $F_{x,y}=0$:

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = fv \qquad -\frac{1}{\rho}\frac{\partial p}{\partial y} = fu$$

For Synoptic Scale Motions:

L ~ 1000 KM ~ 10^6 m
U ~ 10 ms^{-1}
f ~ 7 x 10^{-5} s^{-1}

R_o = U/fL ~ 1/7 ~14%

The smallness of the Rossby number is a measure of the accuracy of the Geostrophic Approximation. At mid-latitudes the geostrophic equations are generally accurate to about 10%.

Geostrophic Equations of Motion

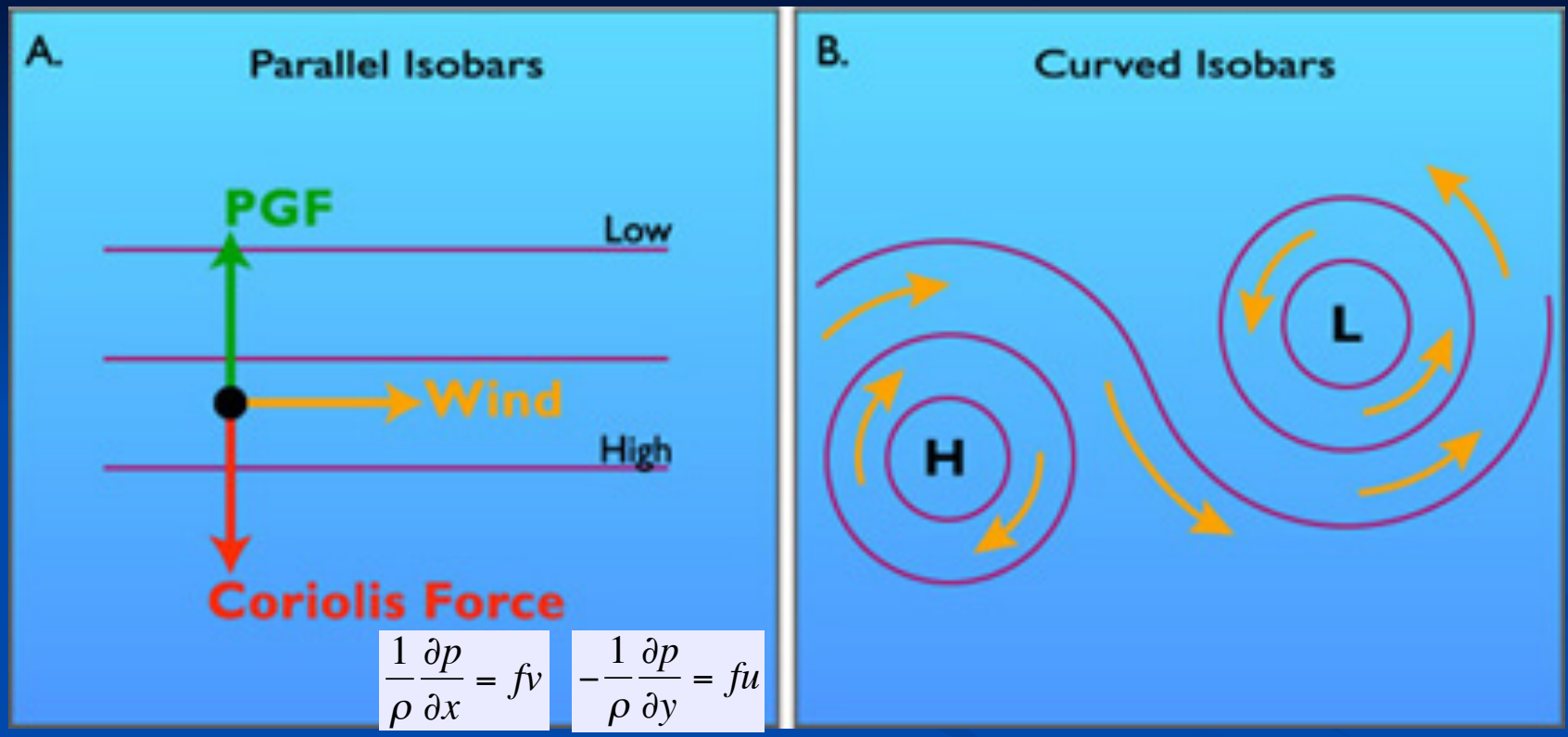
The smallness of the Rossby number is a measure of the accuracy of the Geostrophic Approximation. At mid-latitudes the geostrophic equations are accurate to about 10%

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = fv \qquad -\frac{1}{\rho}\frac{\partial p}{\partial y} = fu$$

These equations of motion are **diagnostic** equations, i.e., they can be used to infer the velocity field if the pressure variation is known, or they can be used to determine the pressure field if the velocity field is known. These equations **cannot** be used to determine the time evolution of either the pressure or temperature fields.

In order to determine the time evolution, the du/dt term is needed.

Schematic View of Geostrophic Balance



$$\frac{1}{\rho}\frac{\partial p}{\partial x} = fv \qquad -\frac{1}{\rho}\frac{\partial p}{\partial y} = fu$$

To first approximation, the horizontal balance of forces in the Earth's atmosphere is between the pressure gradient force and the Coriolis force.

The approximation that they are in perfect balance is known as the **Geostrophic Wind**. It is typically accurate to of order 10% at mid latitudes, away from the equator, for synoptic scale motions.

Friction

The role of friction in the atmosphere is to produce a "drag" on atmospheric motion. However, the magnitude of the frictional drag is generally very difficult to quantify. The reason is that the drag force is due to a variety of physical processes, all of which transfer momentum between the surface and the free atmosphere. The Frictional Force per unit mass:

$$F = -\frac{1}{\rho}\frac{\partial \tau}{\partial z}$$

where τ represents the vertical component of the **shear stress** (rate of vertical exchange of horizontal momentum) due to the presence of smaller, unresolved scales of motion. The **shear stress** at the surface of the Earth is:

$$\tau_s = - \rho C_D V_s \mathbf{V}$$

C_D = drag coefficient, V_s = scaler velocity, V = vector velocity

Effects of Friction

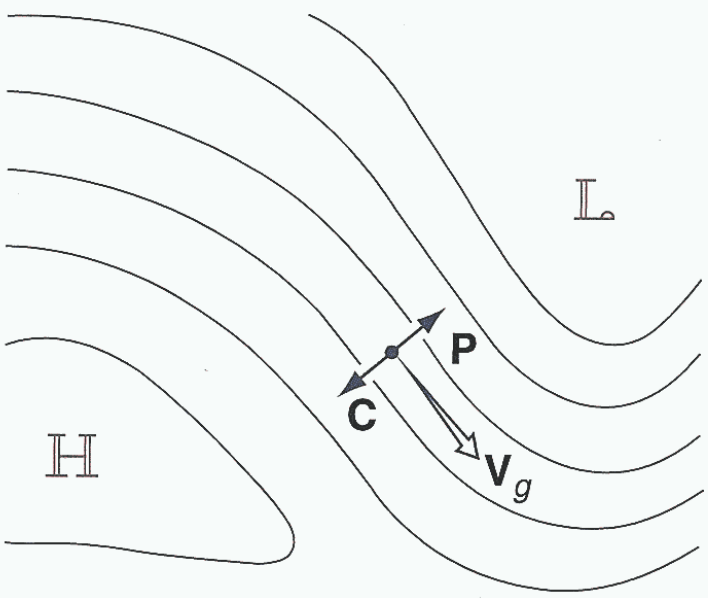


Fig. 7.9 The geostrophic wind V_g and its relationship to the horizontal pressure gradient force P and the Coriolis force C in the northern hemisphere.

The **Coriolis Force** and **Pressure Gradient Force** are always perpendicular to the direction of flow. However, the **Frictional Force** is always opposite to the direction of flow.

Effects of Friction on Flow Direction

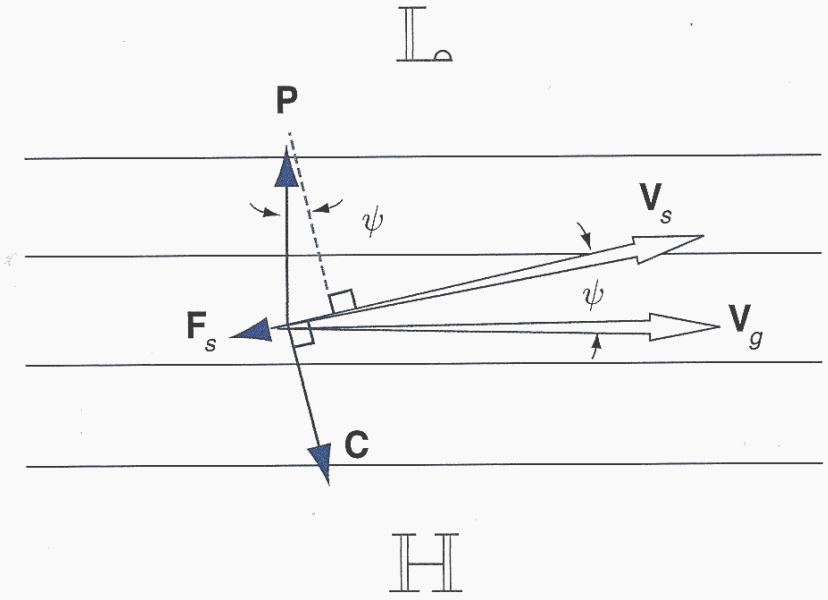


Fig. 7.10 The three-way balance of forces required for steady surface winds in the presence of the frictional drag force F in the northern hemisphere. Solid lines represent isobars or geopotential height contours on a weather chart.

The effect of friction is to slow down the flow. The resulting balance of forces leads to a cross-isobar drift, generally from high pressure to low pressure. Thus friction is a primary means of generating flow which is not parallel to the isobars.

The Gradient Wind

For rapidly rotating flow outside of the surface friction layer, the angular rotation produces a centripetal acceleration. The three-way balance of forces in Gradient Flow (Pressure gradient, Coriolis, and Centripetal) leads to a modification of the force balance equations so that:

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{dy} - fu + \frac{u^2}{R}$$

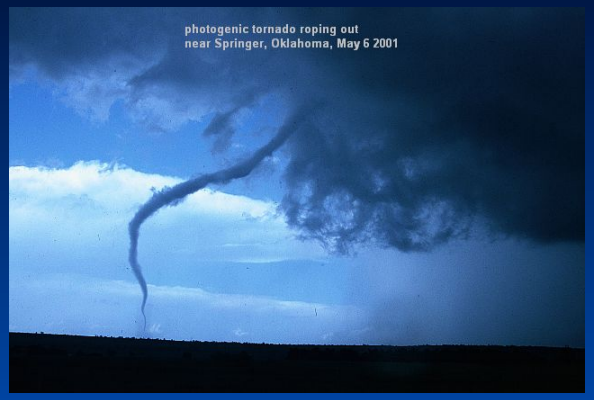
where R is the radius of curvature of the flow streamlines. For steady flow, du/dt = 0, and we can use the definition of geostrophic wind to give an equation that can be solved algebraically for the flow velocity u.

$$0 = u_g - fu + \frac{u^2}{R}$$

Horizontal Balanced Flow

For rapidly rotating steady flow:

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{dy} - fu + \frac{u^2}{R} = 0$$



where R is the radius of curvature of the flow streamlines.

For rapid rotation the pressure gradient force is balanced by the centripetal term. The relative magnitude of the centripetal to the Coriolis term is:



Rossby No. ~ $(U^2/R)/fU \sim U/fR$

Thus the largeness of the Rossby # is a measure of the accuracy of the **Cyclostropyhic Approximation.**

The Three-Way Balance in the Gradient Wind

The wind associated with a three-way balance between pressure gradient, Coriolis, and centrifugal forces is called the gradient wind

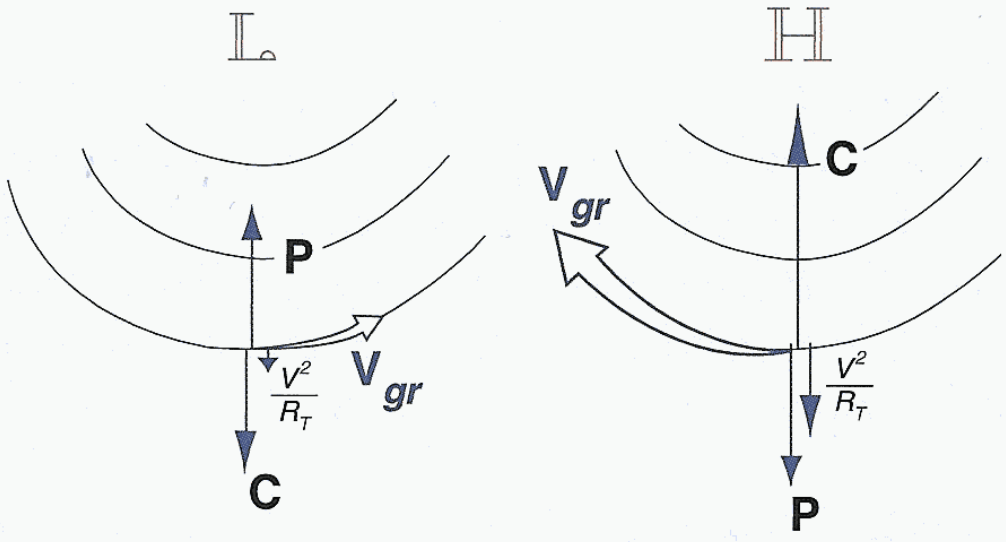


Fig. 7.12 The three-way balance involving the horizontal pressure gradient force P, the Coriolis force C, and the centrifugal force $|V|^2/R_T$ in flow along curved trajectories in the northern hemisphere. (Left) Cyclonic flow. (Right) Anticyclonic flow.

Thermal Wind Equation

The horizontal pressure gradients and geostrophic winds may vary with height in response to horizontal temperature gradients

$$\frac{\partial u_g}{\partial t} = -\frac{g}{fT}\frac{\partial T}{dy} \qquad \frac{\partial v_g}{\partial t} = \frac{g}{fT}\frac{\partial T}{dx}$$

The Thermal Wind Equation implies a diagnostic relationship between the temperature structure and the wind structure.

$$f = 2\Omega \sin\phi$$

Coriolis parameter depends on the latitude ϕ

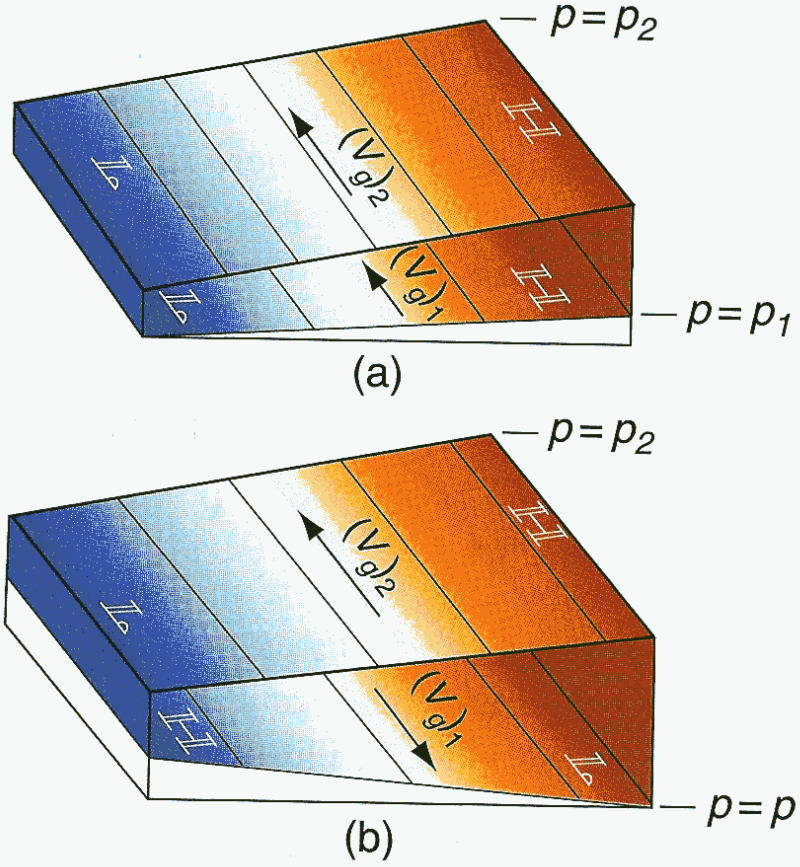


Fig. 7.13 The change of the geostrophic wind with height in an equivalent barotropic flow in the northern hemisphere: (a) V_g increasing with height within the layer and (b) V_g reversing direction. The temperature gradient within the layer is indicated by the shading: blue (cold) coincides with low thickness and tan (warm) with high thickness.

Governing Equations

$$P = \rho_{air} R T_v$$

$$\frac{\partial U_j}{\partial x_j} = 0$$

$$\frac{dU_i}{dt} = \frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\delta_{i3} g + f \varepsilon_{ij3} U_j - \frac{1}{\rho} \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j^2}$$

$$\frac{d\theta}{dt} = \frac{\partial \theta}{\partial t} + U_j \frac{\partial \theta}{\partial x_j} = \nu_\theta \frac{\partial^2 \theta}{\partial x_j^2} + S_H$$

$$\frac{dq_t}{dt} = \frac{\partial q_t}{\partial t} + U_j \frac{\partial q_t}{\partial x_j} = \nu_q \frac{\partial^2 q}{\partial x_j^2} + S_q$$

$$\frac{dC}{dt} = \frac{\partial C}{\partial t} + U_j \frac{\partial C}{\partial x_j} = \nu_C \frac{\partial^2 C}{\partial x_j^2} + S_C$$

Global Circulation
Single-Cell Circulation Model

- Our knowledge of global winds comes from two sources; the patterns of pressure and winds observed worldwide and theoretical studies of fluid motion.
- George Hadley in 735 proposed that the large temperature contrast between the poles and the equator creates one large convection cell in each hemisphere.
- But this did not take into account the fact that Earth rotates.

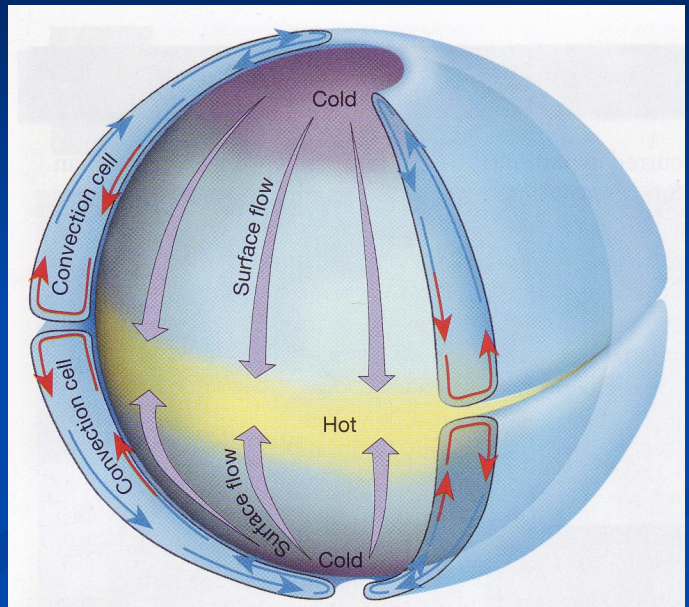
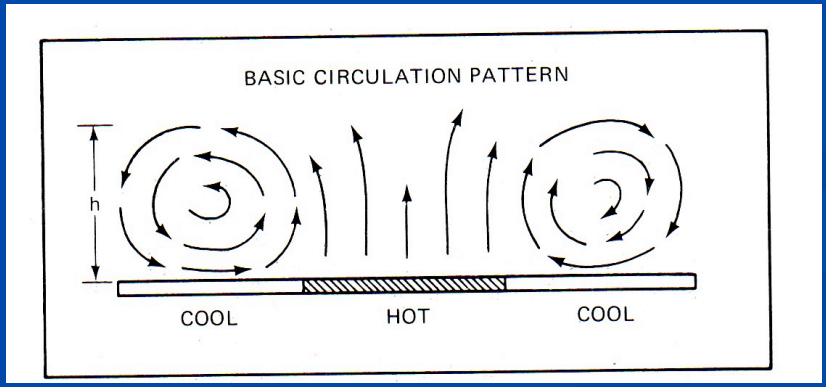
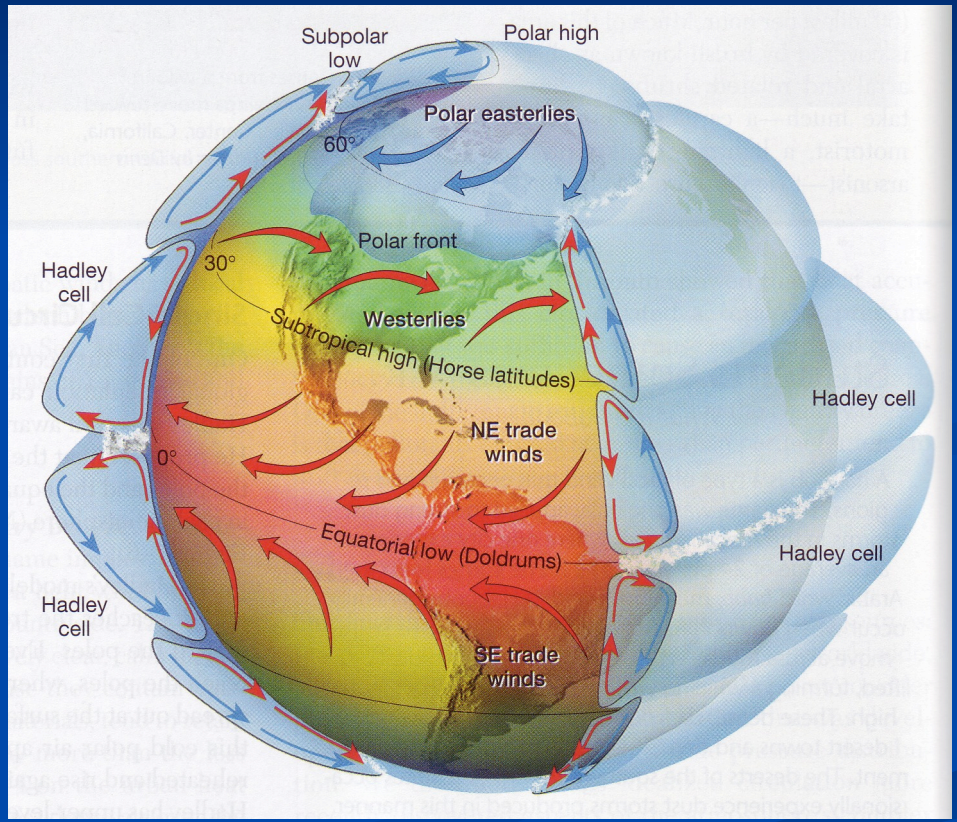


FIGURE 7-6 Global circulation on a nonrotating Earth. A simple convection system is produced by unequal heating of the atmosphere on a nonrotating Earth.



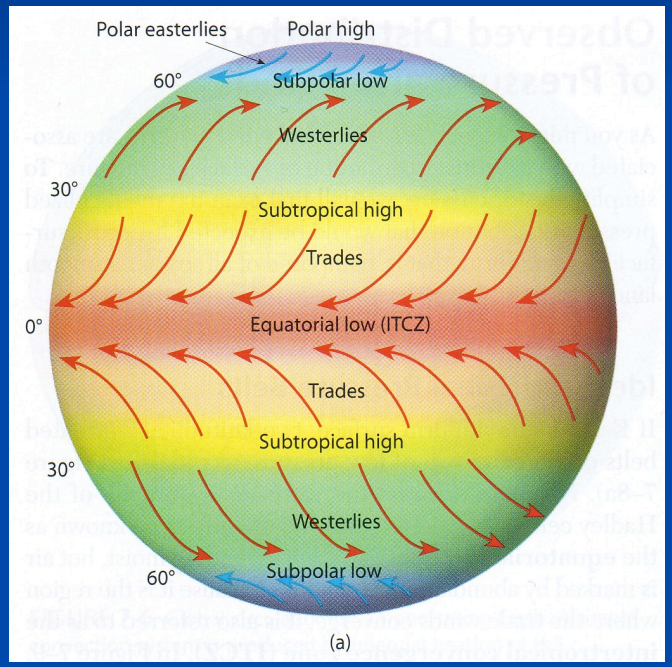
Global Circulation
Three-Cell Circulation Model

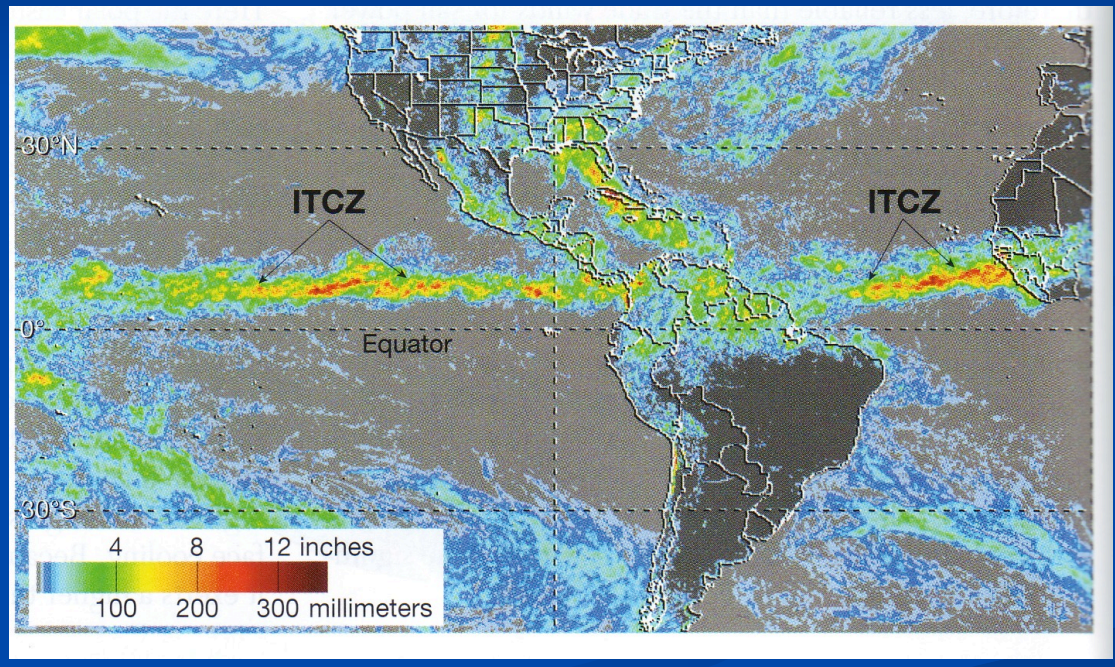
- In the 1920s a three-cell circulation model was proposed that takes into account the fact that Earth rotates.
- Three cells include:
  - Hadley Cell
  - Subpolar low
  - Polar high



Observed Distribution of Pressure and Winds
Idealized Zonal Pressure Belts

- If Earth had a uniform surface, two latitudinally oriented belts of high and two of low pressure would exist.

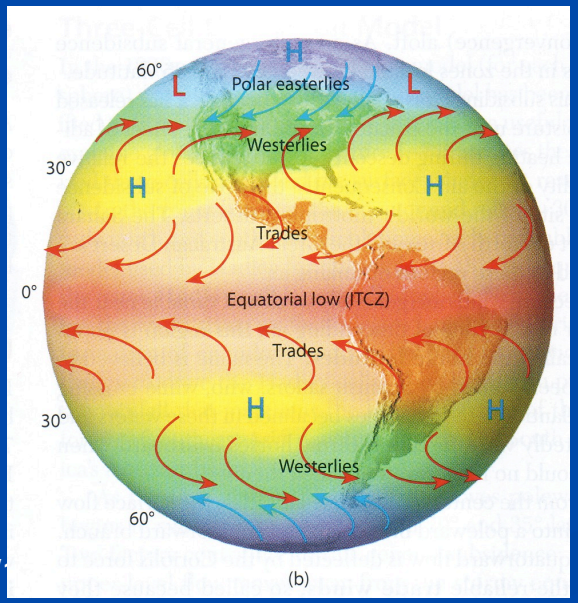


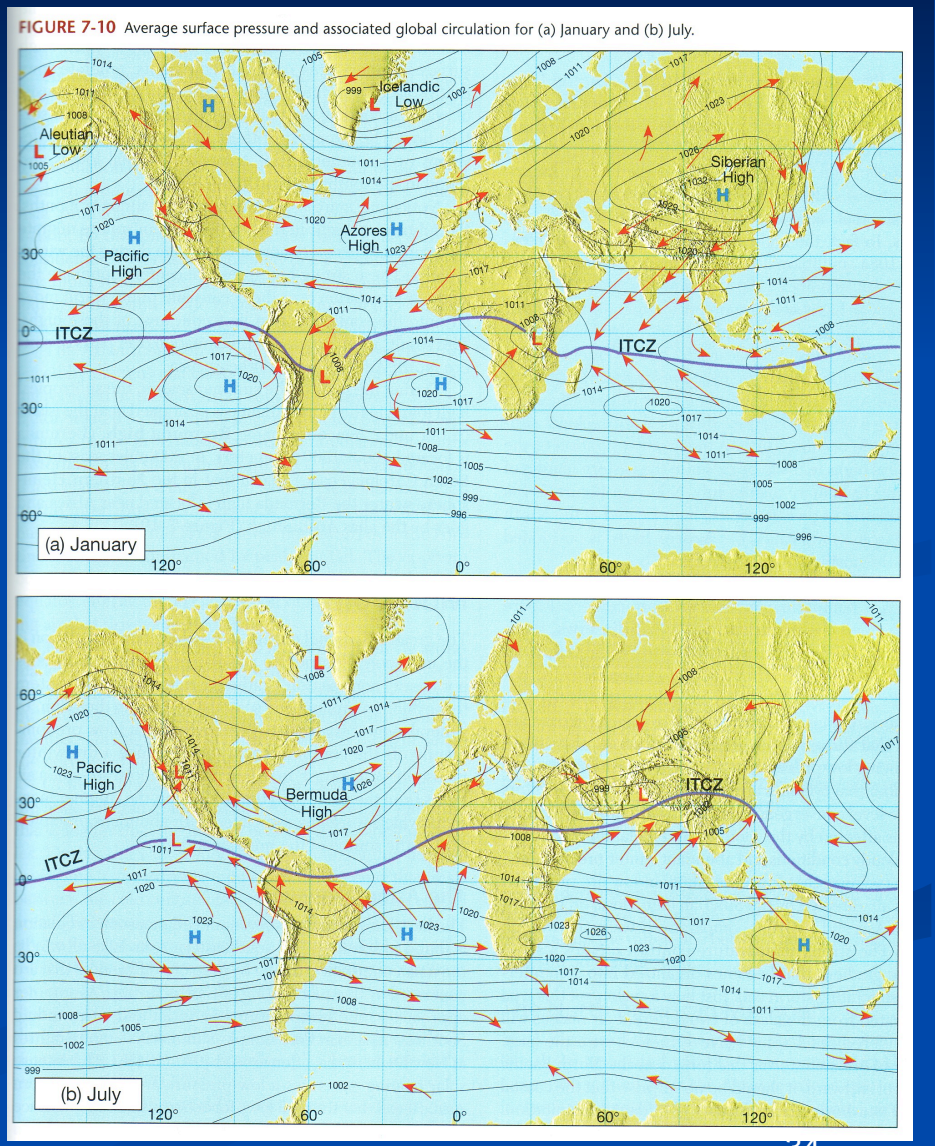


Observed Distribution of Pressure and Winds

Idealized Zonal Pressure Belts

- The real Earth with disruptions of the zonal pattern caused by large landmasses.
- This disruptions break up pressure zones into semipermanent high- and low-pressure cells.





Global Winds & Ocean Currents

- The atmosphere provides coupling between the solid Earth's surface and oceans.
- In this coupling, friction plays an important role (e.g., the surface layer of water to move).
- As a results, winds are the primary driving force of **ocean currents**.

